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OUTLINE of ELECTRICITY

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1955

By

JOSEPH H. HOWEY

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Chapter 1

ELECTRICITY AND MATTER

10. Electricity occurs in many different forms. It appears in spectacular lightning strokes and in the quiet glow of pocket flashlights. It drives locomotives and toasts bread. It is found in atoms and in living cells. Its influence extends everywhere, and everywhere it is the same electricity obeying the same fundamental laws.

Electricity may move from one place to another, or it may remain at rest in a given place. Some of its most fundamental properties appear the same whether it is moving or at rest, and these are referred to as the properties of static electricity. Other properties appear only when the electricity is moving, and these are referred to as the properties of electric currents. Our study of electricity will begin with the static properties. These properties can be most conveniently studied by observing electricity at rest, but the reader should bear in mind that these static properties are possessed whether the electricity is at rest or in motion.

11. Electric Charges Produced by Friction. When certain materials are rubbed together, they acquire a special ability to attract other pieces of material. For example, a fountain pen rubbed on a woolen coat sleeve can pick up pieces of hair, paper, and other small objects. A stick of wax rubbed with wool will show similar attractive powers. So also will a toy balloon rubbed with fur, or a glass bulb rubbed with silk. In general, any body that has acquired an attractive power in this way is said to be charged with electricity, and the forces involved are referred to as electric forces.

A charged body will always attract a neutral body, but two charged bodies may either repel or attract each other depending on their nature. For example, two rubber balloons each rubbed with fur will repel each other. A balloon rubbed with fur will be repelled by a piece of wax rubbed with wool but it will be attracted by a glass bulb rubbed with silk. Two glass bulbs rubbed with silk will repel each other, and so on. In general, it is found that all charged bodies can be divided into two groups depending on whether they behave like a rubber balloon rubbed with fur or like a glass bulb rubbed with silk. Any body in one group will then repel any body in the same group but attract any body in the other group. Thus we say there are two kinds of electric charges, and that like charges repel while unlike charges attract. The kind of charge found on glass rubbed with silk is arbitrarily called positive charge, and the other kind is called negative.

When electrification is produced by rubbing two materials together, charges of opposite sign will always be generated on the two bodies. Thus when a glass bulb is charged positively by rubbing it with silk, the silk gets a negative charge. So also when a rubber balloon is charged negatively by rubbing it with fur, the fur gets a positive charge.

12. Particles of Electricity. The electricity on a charged body is usually more or less distributed over the surface of the body. Such a surface layer of electricity actually consists of a large number of small separate charges, just as a layer of dust is made up of separate particles. Negative electricity occurs in the form of electrons, which are small permanently charged particles of mass. All electrons have the same charge and the same mass and each electron behaves as an indivisible unit. The mass of an electron is 9.11×10^{-28} gm. This is so small that it would require 1800 electrons to weigh as much as a hydrogen atom, which is the smallest atom of all the chemical elements. Positive electricity is most commonly found in the form of permanently charged particles called protons. Each proton has a positive charge equal in magnitude to the negative charge on an electron. The mass of a proton is 1.67×10^{-24} gm, which is approximately equal to the mass of the hydrogen atom. The proton is thus about 1800 times as heavy as an electron.

Electrons and protons may be referred to as sub-atomic particles, since they are smaller and simpler than the atoms of the chemical elements, and since they occur as component parts of these atoms. A chemical atom may be defined as the smallest piece of a elemental material that can exist and still exhibit the characteristic chemical properties of that element. Chemical atoms behave as indivisible units as far as chemical reactions are concerned, but they are really composite bodies containing electrons protons and neutrons. A neutron is another type of sub-atomic particle which has approximately the same mass as a proton but which has no electric charge.

13. The Structure of Atoms. As stated above, an atom is a composite structure. Each atom has at its center a nucleus, which is a closely packed combination of protons and neutrons locked together by non-electric forces. The nucleus accordingly has a positive charge equal to the total charge of all the protons included. Outside this nucleus, an atom has one or more electrons rather loosely held by the electrostatic attraction of the positively charged nucleus. Each atom normally has one electron for each proton inside the nucleus, so that the total charge on the atom is zero.

For introductory purposes an atom may be pictured as a planetary system like our solar system. The nucleus of the atom corresponds to the sun. The electrons correspond to the planets. Electric forces hold the electrons to the nucleus in an atom just as gravitational forces hold the planets to the sun in the solar system. Also the relative sizes and distances in an atom are such that most of the space in an atom is empty space just as it is in the solar system. An atom with an atomic number of five is illustrated in Fig. 13. There are five protons (⊕) and five neutrons (●) in the nucleus, but not all of them show as the figure is drawn. Atoms are far too small to be seen by visible light, and the arrangement of the parts inside the atom must be inferred from indirect experiments. Illustrations of atoms as shown in Fig. 13 are therefore to be regarded as schematic diagrams rather than actual pictures.

The mass of an atom is determined mostly by the mass of its nucleus. This must be true because the mass of an electron is small compared to the mass of a proton. The chemical nature of an atom depends on how many electrons it has outside the nucleus. That in turn depends on the number of protons in the nucleus. Thus it is that the chemical nature depends on the number of protons in the nucleus. That number is referred to as the atomic number of the chemical element involved.

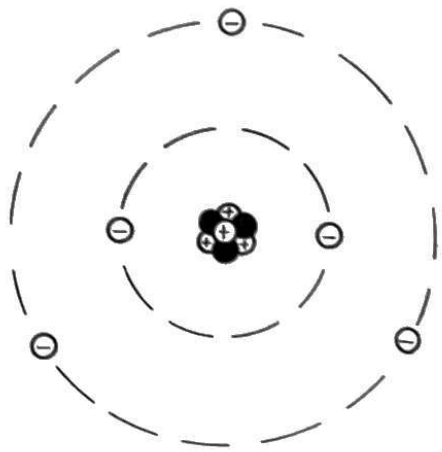

Fig. 13.

14. Extraction of Electric Particles from Atoms. Electrons and protons can be extracted from atoms and left to exist as independent particles unattached to any atom. The electrons come from the outer part of the atom and may be extracted rather easily by any one of several processes. They may be pulled out of atoms directly by applying large electric forces to the surface of a body of material. They may be ejected from the surface of a body by illuminating the surface with certain kinds of radiation, such as ultra-violet light. They may be literally knocked out of the material by bombarding it with high speed particles. Most simply of all, they can be "boiled" out by heating the material hot enough so that the electrons evaporate from the surface much as molecules of vapor evaporate from the surface of a liquid. Confirming the fact that electrons are a universal constituent of all matter, it is found that all electrons are the same regardless of the material from which they are obtained, and regardless of the method by which they are extracted.

Since electrons come from the outer part of an atom, they can be removed without affecting the structure of the nucleus. This means that an atom can lose electrons without losing its

chemical identity. In general, it will later gain back some stray electrons and again become a normal atom of the same kind.

The extraction of a proton from a nucleus is a relatively difficult process, and it can be accomplished only by the application of a large amount of energy. An atom from which a proton has been removed is permanently changed into a different chemical element by the process. Such transmutations will be studied in a later part of this text.

When electrons or protons exist as independent particles, they behave like any other particles of mass, except for the added characteristic of carrying a permanent charge. They have the ordinary property of inertia, and obey the same laws of mechanics as do the larger masses which we are more familiar. Because of their smallness, electrons and protons can be accelerated to very high velocities by the sources of energy at our disposal. These velocities are many times greater than the highest velocities man has been able to impart to ordinary visible objects. In applying the usual laws of mechanics for these high velocities, it must be remembered that the mass of any body may be considered constant only if the velocity is small compared to the velocity of light. Actually the mass m of a body increases with the velocity v according to the equation $m = m_0/\sqrt{1 - (v/c)^2}$ where m_0 is the mass at rest and c is the velocity of light. As long as v is less than 1/20th of c, the difference between m and m_0 is less than .1%.

15. Electronic Explanation of Charging by Rubbing. Since any body of matter is an aggregate of atoms, it must always contain a large number of both negative and positive charges. A so-called uncharged body is electrically neutral, not because of the absence of charge, but rather because the two kinds of charges occur in the atoms in equal amounts. The two kinds of charge then neutralize each other as far as any external effect is concerned.

For a body to acquire an electric charge, it must be left with an excess charge of one kind or another. When two substances are electrified by rubbing, some of the electrons that belong on one body get attached to the other body. This leaves the first body with an excess of positive charge, while the second body carries an equal excess of negative charge.

The ability of a material to take electrons from another material by rubbing is a relative ability. In other words, all materials can be arranged in a sequence such that any given material will gain electrons from materials which precede it in the sequence, but lose electrons to materials which follow it. The forces involved in the transfer of electrons by rubbing are probably similar to those which account for the chemical combinations of atoms in molecules.

It should be noted that strong electrification may occur when two substances are pressed together and then separated quickly without any large scale relative motion between the two surfaces. Thus a belt running on a pulley without obvious slippage may acquire an electric charge. In factories where charges may be generated by moving belts in this way, the resulting sparks may constitute a fire hazard in the presence of explosive dust or vapor. From a microscopic point of view, both rubbing and temporary pressure have a similar effect. Both processes bring large numbers of the two kinds of molecules into close contact, and thus the same electrical effect may be expected when the molecules are separated.

16. Units of Charge. Since electricity occurs in the form of particles which are all the same size, the amount of electricity on one of these particles constitutes a natural unit of measurement for electrical charge. Considered as a unit, the amount of charge on one electron is referred to as the electronic charge and it is represented by the symbol e.

The electronic charge is an inconveniently small unit for many purposes. Other larger units of charge are the statcoulomb and coulomb. The statcoulomb is equal to 2.08×10^9 electronic charges. The coulomb is equal to 6.24×10^{18} electronic charges. A microcoulomb is 10^{-6} coulomb. A number of relationships between these various quantities are given in the table below in the form in which they will probably be most convenient for reference.

$$1 \text{ coulomb (coul)} = 3 \times 10^9 \text{ statcoulomb (statcoul)}$$
$$1 \text{ coulomb (coul)} = 10^6 \text{ micro-coulomb } (\mu\text{-coul})$$
$$e = 16.02 \times 10^{-20} \text{ coul.}$$

The sizes of the coulomb and statcoulomb were arbitrarily chosen with regard to other considerations before the existence of electrons was known, and hence the relationship between the larger units and the electronic charge had to be determined by experiment.

17. Coulomb's Law of Force between Charges. Experiment shows that the mutual force between any two electric charges is directly proportional to the size of either charge and inversely proportional to the distance between the charges. This law of behavior was recognized by Coulomb (1736-1806) and is known by his name. Coulomb's Law of force may be expressed in mathematical form by the equation

$$F = K_0 \frac{qQ}{r^2} \tag{17-1}$$

where F is the force, K_0 is a constant of proportionality, and r is the distance between the two charges q and Q in empty space.

The constant K_0 can be determined by measuring the force between two known charges a given distance apart. Its value is

$$K_0 = 9 \times 10^9 \ \frac{\text{newt}}{\left(\frac{\text{coul}^2}{\text{m}^2}\right)} \tag{17-2}$$

If the measurement is made in air, a slightly different constant should be used. However the difference is so small that it will not affect the first three significant figures that are to be used in problems in this textbook. The units of K_0 as given have been chosen so that the units will balance in Eq. (17-2), if the force is in newtons, the distance in meters and the charges are in coulombs. Equation (17-1) is applicable to concentrated bodies of charge which are small compared to the distance r.

Equation (17-1) shows that it would be impossible to handle two segregated static charges as large as a coulomb within the confines of an ordinary laboratory. The force between two coulombs of charge ten feet apart would be about one hundred thousand tons. The static charges encountered in practice are so much smaller than a coulomb that they can most conveniently be expressed in statcoulombs or microcoulombs. Two charges of a microcoulomb each placed a meter apart would give a force of the order of 1 gm.

Chapter 2

ELECTRIC CHARGES ON MATERIAL BODIES

20. The Electroscope. The presence of electric charges on some bodies can be detected by the changed appearance of the body. For example a piece of fur may show a charge by the way the hairs stand on end. A silk tassel shows a charge by the way the separate strands spread apart from each other. This spreading is caused by the mutual repulsion of similar charges on the different strands.

An electroscope is a device particularly designed to indicate the presence of a charge as clearly as possible. The most common type of electroscope is the so-called leaf electroscope. A leaf electroscope has a narrow strip of very thin metal foil F suspended beside a vertical rod A by fastening the top end of the foil to the side of the rod. The lower end of the foil hangs free at the lower end of the rod. The rod itself is supported near its top end by a non-metallic collar C, as shown in Fig. 20. A cylindrical case G with glass sides protects the delicate foil without obstructing the view. When an electroscope is charged, the mutual repulsion of the charges on

the leaf and rod will make the lower end of the leaf swing away from the rod. The more charge there is, the farther out the leaf will swing. Thus the deflection of the leaf indicates how strongly the electroscope is charged. An electroscope will deflect in the same way whether the excess charges are positive or negative. An electroscope can be charged quite simply by rubbing the top end of the metal rod with a stretched rubber band. It can also be charged by touching it directly or indirectly to a charged body.

Fig. 20.

Experiment shows that an electroscope can be influenced by bringing a charged body near it without actually touching it. In other words, an uncharged electroscope will be deflected merely by the approach of a charge of either sign. If an electroscope is already charged, its deflection will be increased by the approach of a like charge, and decreased by the approach of an unlike charge. Thus an electroscope can be used to detect the sign of unknown charges provided there is a known charge available for comparison.

21. Conductors and Insulators. As explained above, a charged body is one having an excess of either kind of charge. If there are excess charges of either sign, these charges will repel each other according to Coulomb's law, and thus will tend to move away from points where they are crowded together, and toward points where there is less crowding. If the body of matter permits the charges to move from one point to another, the material is referred to as a conductor. If the body of the material does not permit the charges to move from one point to another, the material is called an insulator. There are no perfect conductors since all substances offer some resistance to the flow of charges. Also there are no perfect insulators since all materials will permit at least a little motion of charge. In general, metals are the best conductors of all known substances, while materials such as glass, wax and plastics are relatively good insulators. Other substances such as carbon, moist wood, etc., are intermediate materials which do not conduct as well as the metals, but conduct better than glass and wax.

The properties of any material as a conductor or an insulator can be observed by holding a piece of the material in your hand and touching the material to a charged electroscope. If the material is a good conductor the charges which are on the electroscope will flow off to your hand and the electroscope will be discharged. Observed in this way, the human body itself is a relatively good conductor, because an electroscope will be discharged immediately when touched directly by the hand. Pieces of metal will apparently discharge an electroscope as quickly as when it is touched directly by hand. A piece of wood will discharge the electroscope slowly. Insulators such as glass will discharge the electroscope so slowly that the leakage may not be detected unless the observations are made over a long period of time.

Gases are normally almost perfect insulators. Under some conditions however, a body of gas may become conducting. For example, if a lighted match is held in the air near a charged electroscope, the electroscope will be discharged through the air. Other conditions which make a gas conducting will be mentioned later (Sect. 24).

22. Charging and Discharging Bodies by Conduction. A conducting body can be charged by touching it to a charged conductor. For example, suppose we have an uncharged metal ball B and a charged ball A as shown at (a) in Fig. 22. If B is touched to A as shown at (b), the charges which were crowded on A will spread over A and B. When B is removed from A, it will then carry some of the charge with it.

A conducting body can also be charged by touching it to a charged insulator. Thus a metal ball touched to a charged glass rod will receive some charge. In this case however, the ball will receive charge only from the immediate area of contact, and it will be necessary to slide the ball over a considerable area of the glass surface to get an appreciable charge.

All parts of a conducting body can be discharged by touching it at one end, just as all parts of a tire will be deflated by a hole at one point. On the other hand it is impossible to discharge a charged insulator by touching it at one point. Touching the insulator may remove some charge

from the point of contact, but the charges on the other parts of the body will remain where they are. To discharge an insulator we must accordingly use a process that directly affects every point of the surface. One method of discharging an insulator is to hold it in a flame. The charges generated in the flame then neutralize the charges on the body. This method is used in printing presses where the moving paper may become so strongly charged that it is difficult to handle. To prevent this the paper is sometimes passed over a row of burning gas jets.

Since a conducting body can be discharged by touching it at only one point, such bodies must be carefully mounted on insulating supports if we wish to have them hold a charge. In practice, static charges produced by rubbing are not often found on metals, because the charges usually leak off of the metals as fast as they are generated. Large static charges can be generated on metals provided they are well insulated.

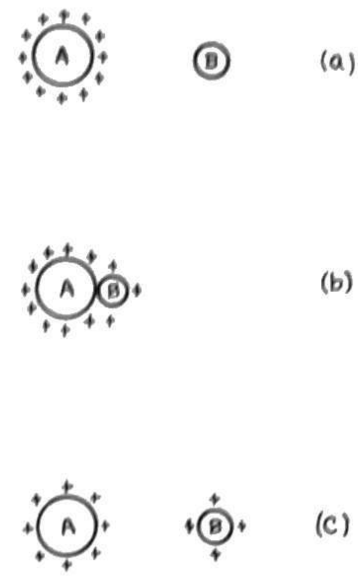

Fig. 22

23. <u>The Distribution of Charge on Conductors</u>. It follows from the properties of a conducting material that there can be no excess static charge of either sign distributed throughout the <u>interior</u> of a conducting body. Under the forces of mutual repulsion, any such charges will move as far apart as possible and come to rest only when they have reached the surface of the conductor. For the same reason there will be no charge on the inner surface of an empty hollow metal body.

The distribution of charge on a hollow conductor may be demonstrated by an experiment having three successive steps (a), (b) and (c) as indicated in Fig. 23-1. This experiment shows that a metal ball B momentarily touched to the inside of a hollow charged conductor A can be removed without having received any charge. On the other hand, if the metal ball is touched to the outside of the charged conductor, and then removed, it will have received a charge as indicated in Fig. 23-2.

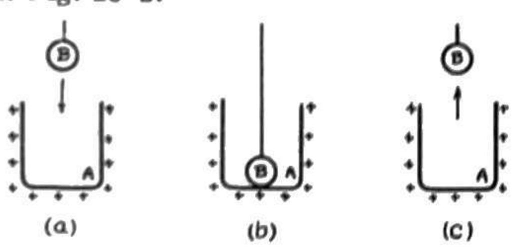

Fig. 23-1

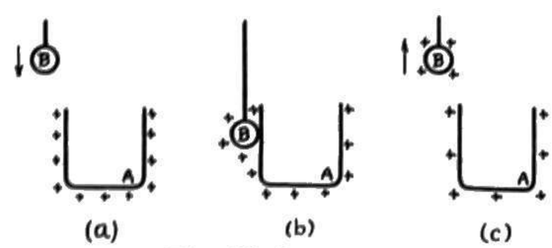

Fig. 23-2

24. <u>The Mechanism of Electrical Conduction</u>. Any material which contains charges that are free to move will act as a conductor. In solid bodies where the atoms are rigidly fixed in space, conduction apparently depends on the motion of temporarily detached electrons moving in the spaces between the atoms. The number of electrons per unit volume that are detached from the atoms at any particular time is different for different solids, and this accounts for differences in conduction. Metals are distinguished by having a relatively large number of these so-called free electrons, and for this reason they are the best conductors.

In some liquids, conduction may be due to the motion of free electrons as it is in solids. In other liquids, atoms which have acquired a charge of either sign may move among the uncharged atoms of the liquid, thereby constituting a flow of charge.

Gases act as conductors when some of the molecules become charged by losing or gaining electrons. These mobile charged particles which make the gas conducting are called ions. The atmosphere always contains a few such ions which have been produced by cosmic radiation.

Additional ions can be produced by subjecting the gas to x-rays or radioactive radiations. Chemical combustion also produces ions and such ions exist in the air near open flames.

Taking all kinds of conductors into account, a certain observed conductive effect may be due to positive charges moving in one direction, negative charges moving in the other direction, or to both positive and negative charges moving simultaneously in opposite directions through the same conductor. From any of the effects of conduction which may be observed outside the conductor, it is generally impossible to tell which kind of charge is moving inside the conductor. Following historical precedent, we shall accordingly discuss all such effects on the assumption that they are due to the motion of positive charge in the conductor. We will not concern ourselves with the truth of this assumption except when we wish to specifically consider the events which occur in the hidden interior of some particular type of conductor. It will also be the general practice in this book to use the term charge to mean a positive charge, unless specifically designated otherwise.

Chapter 3

ELECTRIC FIELDS

30. Electric Field Strength. We have seen that a given fixed charge Q can exert forces on other charges even if they are some distance away. Hence the influence of a charge Q must extend out into all the space around it, becoming weaker as the distance from Q increases. In general if there is such an electric influence acting at a point in space, there is said to be an electric field at that point. The strength $\mathcal{E}$ of this field is defined as the force per unit charge which will act on a movable charge q placed at the point in question. Hence, by definition

$$\mathcal{E} = F/q \tag{30-1}$$

For example, suppose that we wish to measure the field strength at a certain point, and we find that a force of 10 newtons acts on a movable test charge of 2 microcoulombs placed at that point. The field strength will accordingly be given by

$$\mathcal{E} = \frac{10 \text{ newtons}}{2\ \mu\text{-coul}} = 5 \text{ newt per } \mu\text{-coul}$$

It follows from Coulombs law that the force on any charge is proportional to the charge. Hence after $\mathcal{E}$ has been found by measuring the force on any one charge, the force that will act on any other charge placed at the same point may be computed by writing Eq. (30-1) in the form

$$F = \mathcal{E}q \tag{30-2}$$

For example, if $\mathcal{E}$ is 2 newtons per microcoulomb as given above, the force on a charge of 50 μ-coul would be given by

$$F = 2\ \frac{\text{newt}}{\mu\text{-coul}} \times 50\ \mu\text{-coul} = 100 \text{ newt}$$

Note that although the observed field at any point must depend on one or more fixed charges near by, we can measure the field by experiment without necessarily knowing anything about the fixed charges which produce it.

Electric field strength is a vector quantity, and its direction is taken by definition to be the direction of the force on a positive test charge. The force that will act on a negative charge placed in an electrical field is always opposite to the force that would act on a positive charge placed at the same point.

31. Units of Electric Field Strength. It follows from the definition of electric field strength as given in Eq. (30-1) that any unit of force divided by any unit of charge may be used as a unit of field strength. Thus in the examples quoted in the preceding section, the field strength was expressed in newtons per microcoulomb. Another unit that can be used is the newton per coulomb. A simple transformation of units shows that

$$\frac{1 \text{ newt}}{\mu\text{-coulomb}} = 10^6 \ \frac{\text{newt}}{\text{coulomb}}$$

32. Computation of an $\mathcal{E}$ due to known Q's. If we know the location and magnitude of a fixed charge Q which exerts a field $\mathcal{E}$ at a point P, then the field can be computed without necessarily making a measurement of the force F acting on a test charge q. According to Coulomb's law,

$$F = K_0 \frac{qQ}{r^2} \tag{32-1}$$

where r is the distance from Q to the point where q is located. By substituting this computed value of F in the equation $\mathcal{E} = F/q$ we get

$$\mathcal{E} = K_0 \frac{Q}{r^2} \tag{32-2}$$

and thus we have an expression for the field acting on q due to Q.

If there are several fixed charges Q_1, Q_2, etc., which all act on q at the same time, the resultant force on q will be the vector sum of the separate forces. Hence the resultant field strength will be the vector sum of the separate field strengths. This may be expressed by the vector equation

$$\mathcal{E} = K_0 \left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2} + \dots \frac{Q_n}{r_n^2} \right) \tag{32-3}$$

33. The Graphical Representation of an Electric Field. Lines of Force. Since electric field strength is a vector quantity, it has both a magnitude and a direction. Thus the value of $\mathcal{E}$ at any one point in space can be represented by a single arrow having a definite length and direction. However, if we wish to represent the field strength at every point in a given region, there would be an infinite number of arrows required, because there are an infinite number of points, and $\mathcal{E}$ will have a value at each point. It is therefore found convenient to represent the values of $\mathcal{E}$ throughout a given region by drawing continuous "lines" which every where have the direction of $\mathcal{E}$. These continuous lines can also be used to indicate the magnitude of $\mathcal{E}$ at any point by drawing enough lines to make the magnitude of the field numerically equal to the number of lines drawn per unit perpendicular area at the point in question.

To illustrate the use of these so-called "lines of force" let us show how they can represent the field in the region around a positive point charge Q. Here the field is directed radially away from Q in all directions, and the value of $\mathcal{E}$ at any point a distance r from q is $K_0 \, Q/r^2$. To determine how many radial lines of force must be drawn away from Q to represent the magnitude of the field, let us consider a spherical surface of radius r with its center at Q. The radial lines of force will pass perpendicularly through this spherical surface, and hence we need to draw enough lines so that the number passing through the sphere per unit area is equal to $K_0 \, Q/r^2$. The area of the spherical surface is $4\pi r^2$, and hence the total number of lines will be $K_0 \, 4\pi Q$. Since the radius r does not appear in this expression, the same number of lines must pass through any spherical surface that has its center at Q. This number of lines radiating uniformly from Q in all directions will therefore represent the field at any distance from Q. When r is larger, the same number of lines will be spread farther apart with less lines per unit area passing through a spherical surface. This corresponds exactly to the smaller value of $\mathcal{E}$ at the larger distance.

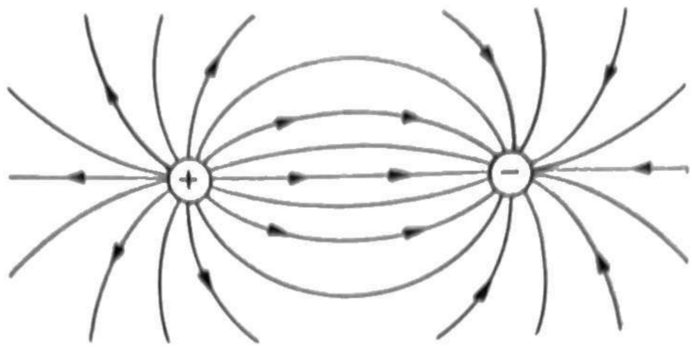

Fig. 33

In general, the electric field due to a positive charge will always be directed away from the charge while that due to a negative charge will be directed toward the charge. Hence lines of force will in general diverge from positive charges and converge to negative charges as shown in Fig. 33. This figure shows the resultant field between a positive charge and a nearby negative charge.

Coulomb's law (Sec. 17) implies that the electrostatic force on a test charge cannot change abruptly in empty space. Hence lines of force can never begin or end at points in empty space. It follows that the lines of force which diverge from a positive charge must continue outward indefinitely unless there is a negative charge on which they can come to an end.

Chapter 4

ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

40. In many important applications, electric charges are used to carry energy. For example, charges moving along a transmission line may carry energy from a power house to a factory or a residence. When this energy reaches its destination, it may be converted into mechanical work by electric motors, or into useful heat or light by various electric devices. Each charge receives energy as it passes through the generator at the power house, and each charge gives out some energy as it passes through the place where energy is expended. Thus the amount of energy that is carried per unit charge is of great practical interest. As we shall see later, it also is a quantity which occupies a most important place in the theory of electricity.

41. Definition of Electric Potential. An electric charge will possess energy if it is in a place where an electrostatic force acts on it. This follows because the charge could do work if it moved under the action of this force. This type of energy is referred to as electrostatic potential energy. It corresponds to the more familiar gravitational potential energy which a material body may possess, except that electrostatic forces are involved in place of gravitational forces.

The amount of electrostatic energy per unit charge possessed by a positive charge q at a given point A is referred to as the electric potential at A. Electric potential is usually represented by the symbol V, with a subscript to indicate the point for which this value applies. Thus the definition of the electric potential V_A at a point A may be written in the form

$$V_A = \frac{\text{Electrostatic energy of q at A}}{q} \tag{40}$$

42. Units of Potential. Any unit of energy divided by any unit of charge may be used as a unit of electric potential. A joule per coulomb is the most commonly used unit, and it is referred to briefly as a volt.

To illustrate the use of units, let us compute the electric potential V_A at a point where a charge of 3 coulombs has an electrostatic potential energy of 45 joules. According to the above definition of potential,

$$V_A = \frac{45 \text{ joules}}{3 \text{ coul}} = 15 \text{ joule per coul, or 15 volts.}$$

As another example, let us compute the energy of a charge of 60 coulombs at the same point. Using Eq. (40) in the form

$$\text{Energy of } q = q\, V_A \tag{42}$$

we get

$$\text{Energy} = 60 \text{ coul} \times 15 \frac{\text{joules}}{\text{coul}} = 900 \text{ joules}$$

43. Potential due to Fixed Point Charges. As explained above, a movable charge will possess electrostatic potential energy if it is in a position where electrostatic forces are acting on it. This implies that there must be other charges placed somewhere near to produce this field. For example if the movable charge q is acted upon by a field $\mathcal{E}$ as indicated in Fig. 43-1, this field might be due to a fixed positive charge Q as shown. Under this condition, we would say that the potential of q was due to the fixed charge Q. The closer q is to the fixed charge, the more work it can do as it moves away under the action of the repulsive force from Q. Starting with Coulomb's law, it can be shown by calculus (See Appendix A) that the potential V_x of a movable charge q placed at a point x a distance r from a fixed point charge Q is given by the equation

Fig. 43-1

$$V_x = K_0 \frac{Q}{r}. \tag{43-1}$$

This equation shows that V_x is greater when the distance r is less, in agreement with what was said above.

Equation (43-1) assumes that the potential energy of a movable charge is zero when it has moved so far from all fixed charges that they no longer exert any appreciably force on it. This remote position is briefly referred to as being at infinity. This equation also implies that a negative fixed charge will produce negative potentials because V will be negative if Q is negative. This is consistent with the fact that a movable positive charge near a fixed negative charge will have less energy than it would have at infinity. In other words, work would have to be done on the positive charge to move it to infinity against the attractive force of the fixed negative charge.

If there are several fixed charges near a point x, each charge exerts its own force just as it would if the others were not there. Hence the total potential at x will be the sum of the potentials due to the separate charges according to the equation

$$V = K_0 \frac{Q_1}{r_1} + K_0 \frac{Q_2}{r_2} + \ldots K_0 \frac{Q_n}{r_n} \tag{43-2}$$

Here the separate potentials from the respective charges are to be added with due regard for the algebraic sign of the charges, but without regard to direction. Potential is like energy in that it is not a vector quantity.

A potential value measured relative to a zero value at infinity is called an absolute value of the potential. Since it is impractable to measure potentials relative to this zero value at infinity, the potential of the earth is usually taken as a convenient zero value. In this respect the measurement of potential resembles the measurement of temperature. There the melting point of ice is often used as a practical zero rather than the less accessible absolute zero of temperature.

44. Potential of a Charged Body. If a number of point charges are distributed over a body A making a total charge Q_A, the resulting potential for any point on the body may be found by adding the potentials due to the separate point charges as indicated in the preceding section. A

simplified expression for the resultant potential in terms of the total charge Q_A can be written in the form

$$V_A = aQ_A \tag{44-1}$$

where a is single factor including the effect of the geometrical distribution of the separate point charges and the units used. Any point on a small charged body will be relatively close to all the charges on the body and the potential will be relatively high for a given total charge Q_A. Thus the factor a is larger for a smaller body.

In general a body A may be at a high potential either because of a charge Q_A on the body itself, or because of a charge Q_B on some nearby body B, or both. This more general situation can be described by the equation

$$V_A = aQ_A + bQ_B \tag{44-2}$$

where the added term bQ_B represents the contribution from the nearby charge Q_B. The factor b is a factor depending on the distribution of the charge on B and on its distance from the body A. If the body B is relatively close to A, the effect of a given Q_B will be large, and thus the factor b will be relatively large.

Equation (44-2) may be applied to an electroscope A charged with a positive charge Q_A as illustrated in Fig. 44-1. Let us assume there is a negative charge Q_B which is at first so far removed from A that it has no appreciable effect on A. In other words, it is so far away that the factor b in Eq. (44-2) is zero. The potential V of the electroscope is then positive due to the charge Q_A on the electroscope itself. If Q_B is now brought closer to the electroscope, the factor b will increase and Q_B will begin to make a negative contribution to the total potential. As the negative term bQ_B increases the resultant potential V will decrease, and the deflection of the electroscope will decrease. By bringing Q_B near enough, the term bQ_B can be made equal in magnitude to the constant term aQ_A. The resultant potential will then be zero, and the deflection will be zero. If Q_B is brought still nearer, the negative term bQ_B will become greater than aQ_A. The resultant potential of the electroscope will then be negative, and the leaf will be deflected again.

Fig. 44-1

If a charged electroscope is approached by another charge of the same size, the magnitude of the resultant potential will increase from the start, and so will the deflection. Thus it is that the sign of an unknown charge can be found by an electroscope if a known charge is available for comparison as mentioned in Sect. 20.

It should be noted that the deflection of an electroscope depends on the potential of the electroscope rather than on the total charge on the electroscope. This is well demonstrated by the fact that an uncharged electroscope can be deflected by a nearby charge which raises the potential of the electroscope without transferring any charge to the electroscope.

45. Potential Difference. The gain (or loss) in electrostatic potential energy per unit charge when a charge moves from one point A to another point B is referred to as the potential difference between A and B. According to the principle of the conservation of energy, the potential difference between two points must be equal to the work done per unit charge by electrostatic forces as a charge moves from one point to the other. The symbol V without any letter subscript will be used here to represent difference in potential. Letting W represent the amount of work involved, and q the charge which is moved, we may therefore write

$$V = W/q \qquad (45\text{-}1)$$

Difference in potential is measured in the same units as potential at a point. For example let us again consider the transfer of energy from a generator G in a power house to a motor M in a factory as shown in Fig. 45. The two parallel lines represent the transmission wires running from the power house to the factory. In the power house, the generator does work on electric charges by pushing them through the generator from A to B. Thus it acts as an electric "pump" to circulate the charges around in the conducting path ABCDA. Because of the work done by the generator, each charge comes out of the generator at B with more energy than it had when it entered at A. If the generator gives a potential difference of 120 volts (i.e., 120 joules per coulomb), then each coulomb comes out with 120 joules more energy than it had before. As the circulating charges pass through the motor, they expend electric energy and come out at D with less energy than they had when they entered the motor at C. This electric energy lost in the motor is converted into mechanical work. If there is no loss in energy along the transmission wires, and if the charges are circulating at a uniform rate, each charge will give out as much energy in the motor as it received from the generator. In other words, the potential difference expended in the motor is equal to that produced by the generator.

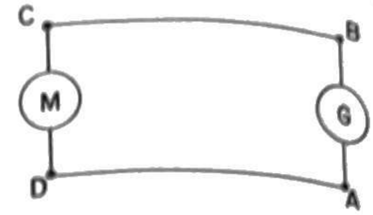

Fig. 45

A given potential difference does not necessarily mean that one of the potentials is zero. In the motor-generator circuit discussed above, the charges returning to from the motor to the generator may still have some energy left. For example, each coulomb might return to the generator carrying 1000 joules provided it had started out from the generator with 1120 joules. That would give the same stated difference of 120 joules per coulomb. In general, if V_B is the potential at one point B, and V_A is the potential at another point A, the gain in potential V in passing from A to B will be given by

$$V = V_B - V_A \qquad (45\text{-}2)$$

46. The Measurement of Potential Difference. Electrometers. The difference in potential between two conductors can be measured according to its definition by allowing a known amount of charge to flow through a conducting path from the point of higher potential to the point of lower potential, and measuring the amount of energy given out per unit charge in the form of heat. A more detailed explanation of this method will be given in Sect. 100.

Differences in potential may also be measured by an electroscope which is provided with a calibrated scale. The deflection of the leaf will then give a direct reading of the difference in potential between the electroscope and its case. An electroscope equipped with a calibrated scale is called an electrometer, or electrostatic voltmeter. A similar, more rugged, meter can be made by using a heavier metal leaf mounted on a pivoted bearing. Portable electrostatic voltmeters constructed in this way are commercially available.

Unless special care is taken to support the case of an electrometer on a better insulator than wood, the case will be effectively in contact with the ground. The potential indicated by the electrometer will then be the difference in potential between the leaf and the ground.

Potential differences can also be measured by meters which indicate the potential difference by indicating the rate of flow of charges from high to low potential through the meter. (See Sect. 97.)

Chapter 5

THE POTENTIAL-FIELD RELATIONSHIP

50. Potential Difference and Electric Force. Electrostatic energy is expended as a charge moves from higher to lower potential along a path between two points. It therefore follows that work is being performed by an electrostatic force. If F is the average component of the electrostatic force on q along the path, and if s is the length of the path, then the energy expended (or work done) will be given by

$$W = Fs \tag{50-1}$$

Combining equations (45-1) and (50-1) gives

$$Vq = W = Fs \tag{50-2}$$

The equation in this form serves to point out that if a point A is at a higher potential than a point B, there must be an average electrostatic force F on a positive charge along the path directed from A to B. Conversely, if there is an electrostatic force along a path between two points, there must be a difference in potential between the two points. A working concept of potential difference should include both the idea of a net gain or loss of energy, and the idea of a force acting along the path, since the two must always exist together.

It follows from the above that if a positive charge is unsupported in an electrostatic field, it will move in the direction of the existing force, that is, from points of high potential to points of low potential. A negative charge will, of course, move in the opposite direction.

51. Potential Gradient. Equation (50-2) from the preceding section can be written

$$F/q = V/s \tag{50-3}$$

Now F/q is the field strength $\mathcal{E}$, and hence we may write[1]

$$\mathcal{E} = V/s \tag{50-4}$$

The quotient V/s is the difference in potential per unit distance along a path, and it is often called the potential gradient.

It follows from Eq. (50-4) that a unit of potential difference divided by a unit of distance can be used as a unit of field strength. For example, a volt per meter is a commonly used unit. It is equivalent to a newton per coulomb as can be seen by writing

$$\frac{1\text{ volt}}{1\text{ meter}} = \frac{1\text{ joule}}{1\text{ coul-meter}} = \frac{1\text{ newton-meter}}{1\text{ coul-meter}} = \frac{1\text{ newton}}{1\text{ coul}}.$$

52. Equipotential Surfaces. If a test charge q moves along a path which is perpendicular to the direction of the electric field at that point, no work will be done on or by the charge. Hence the potential of the charge will remain constant as long as its motion is restricted to a surface perpendicular to the field. Any geometrical surface for which the potential is the same at all points is called an equipotential surface. It follows from above that all equipotential surfaces must be shaped so that they are everywhere perpendicular to any lines of force which pass through them. Thus in the region around a fixed point charge Q from which lines of force diverge radially, any spherical surface with its center at Q would be an equipotential surface.

53. Equipotential Conducting Bodies. It was pointed out above that if there is a potential difference between two points, there must be an electric force acting between the points. If this condition exists on a conductor, positive charge will move from the high potential point to the low potential point, and this transfer of charge will continue until the potential at all points is

1. This assumes that $\mathcal{E}$ is parallel to the path along which the charge moves.

equalized. Thus when the charges come to rest on a conductor, they must necessarily be distributed so that all parts of the conductor are at the same potential. This is a very important principle and it applies to any connected system of conducting bodies as well as to a single conductor.

Since differences in potential and electric field must always exist together, it follows that there can be no electric field along any path through a conductor if the charges have come to rest. Thus no lines of force representing a resultant field can pass through a conductor when the charges are at rest.

54. Electric Fields around Charged Conductors. It follows from above that a static positive charge on a conducting body will be spread over the whole body and will not come to rest until all parts of the body are at the same potential. If a small test charge is used to investigate the field around this body, it will obviously be repelled away from the surface at all points around the body. The lines of force which show the field in the surrounding region will be lines which start from the charges on the surface and extend outward into space. Lines of force and equipotential surfaces must always be mutually perpendicular (Sect. 52) and hence the lines of force from a charged conductor must leave the surface perpendicularly.

If a conducting body is negatively charged, the situation will be the same except that the lines of force will be directed in toward the surface of the body. An illustrative diagram in Fig. 54-1 shows the observed configuration of the lines of force between a positively charged conducting sphere and a negatively charged conducting plane. In all such configurations, it is to be noted that lines of force can never end except at points where there are charges (See Sect. 33).

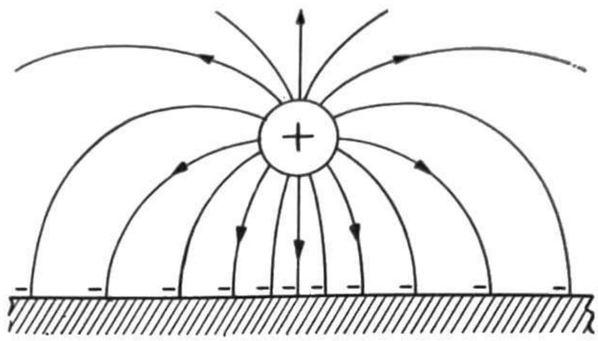

Fig. 54-1

The distribution of charges needed on a conductor to bring all parts to the same potential often requires that the number of charges per unit area will be different for different parts of the surface. In general the density of charge will be greater where a surface has a sharper convex curvature. Thus a body with a sharp point as shown in Fig. 54-2, will have a relatively high concentration of charge on the surface of the point. The field just outside the point will also be higher than it is outside the more gently curved parts of the same conductor.

The presence of a sharp point on a conducting body limits the potential to which the body can be charged. At a certain critical value of the field outside a conductor, the charges on the conductor will begin to leak off of the body into the air. Thus as the potential of a body is raised, leakage will occur sooner if there are sharp points to give a relatively high field. Such leakage can be observed in a dark room as a slight glow at the tip of the point.

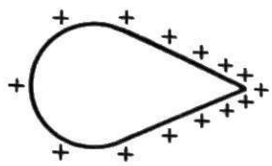

Fig. 54-2

55. The Field between Parallel Plates. In dealing with electricity we will encounter a number of situations involving the electric field between two oppositely charged metal plates as shown in Fig. 55. Let us assume that the plates have been charged by removing a charge + Q from the one plate and placing it on the other. The one plate will then have a charge + Q while the other will be left with an equal negative charge - Q. Let us also assume that the distance between the two plates is small compared to the distance across the face of either plate. Under these conditions, the charges on the two plates will come to rest spread almost uniformly over the inner surfaces of the two plates as shown. This distribution results because the mutual attraction between the two charges keeps them from spreading around to the outer faces of the plates.

The lines of force representing the field between the two plates of Fig. 55 will thus be uniformly spaced and perpendicular to the plates except at points near the edge which we will not consider. The field will obviously be directed from the positive plate toward the negative plate, since a movable charge +q placed between the plates will be repelled by the positive plate and attracted by the negative plate. By using Eq. (32-3) from Sect. 32 to compute the field due to all the charges spread over both plates, we find[1] that the field $\mathcal{E}$ in empty space between the two plates will be given by the equation

$$\mathcal{E} = K_0\ 4\pi Q/A \qquad (55\text{-}1)$$

Here A is the area of either face over which the charge Q is distributed.

A movable charge q placed in the space between the two plates of Fig. 55 will be acted upon by the field that is between the plates. The force F of the field on this movable charge, will be given by the equation $F = \mathcal{E}q$ from Sect. 30. The field directed from the positive plate to the negative plate implies that there will be a difference in potential V between the two plates. If s is the distance between the plates, it follows from Sect. 51 that

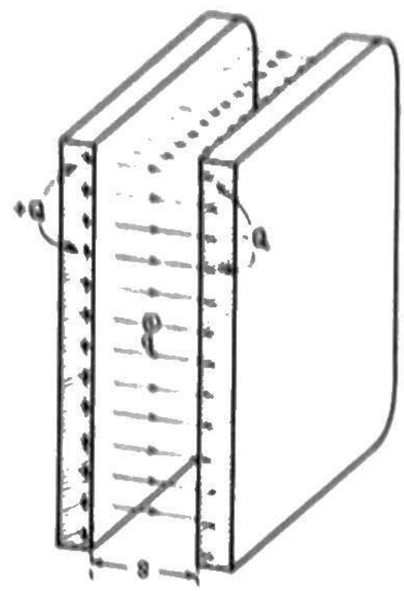

Fig. 55

$$V = \mathcal{E}s \qquad (55\text{-}2)$$

and that

$$Fs = qV \qquad (55\text{-}3)$$

56. The Measurement of the Charge on an Electron. As stated in Sect. 16, the size of the coulomb was arbitrarily chosen before anything was known of the existence of electrons. Thus the value of the electronic charge in terms of the coulomb had to be determined by experiment. One method for measuring the charge on an electron was brought to a high degree of perfection by R. A. Millikan in what is known as his oil-drop method. In this method, charged oil drops are supported in a vertical electrical field between two horizontal metal plates. The potential difference required between the plates to support a given drop is found by trial. The total charge on the drop and the number of electrons involved can then be found as will be explained below.

The charged oil drops used in Millikan's method are very small drops. They are usually obtained in an inclosed space between the horizontal plates by spraying a fine mist of oil above the plates. Some of the drops will then fall through a hole in the upper plate as shown in Fig. 56. The spray must be so fine that the drops almost float in the air. Between the plates such drops will fall so slowly that they can be observed through the microscope M for a considerable period of time. Because of the viscosity of the air, the drops will fall with a uniform terminal velocity which depends on the size of the drop. The smaller the drop, the more slowly it falls. One particular drop must be chosen for observation, and the weight of this drop must be found. The weight of the drops used is too small to be determined by any direct measurement, but it can be determined indirectly for any one drop by observing the velocity with which it falls. Once the velocity is known, the weight w can be computed from a known relationship involving the density of the

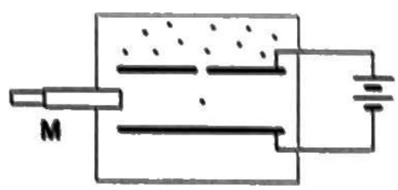

Fig. 56

1. This computation can be made using integral calculus.

drop, and the viscosity of the air.[1] This relationship is based on a theoretical law which was developed by Sir George Stokes (1819-1903) many years before it was used by Millikan.

Most of the drops acquire static charges either from the friction of the spraying process or from the charged particles which are always present in the air (Sect. 24). Any drop that does not have a charge will sooner or later acquire one, particularly if the air between the plates is irradiated with x-rays. The amount of charge on any particular drop under consideration can be determined as follows. A known potential difference V is applied between the horizontal plates to give a vertical electric field between the plates. The direction and magnitude of the potential difference is then adjusted by trial until the oil drop under observation neither moves up or down. This means that the known weight of the drop has been exactly balanced by an upward electric force F on the charged drop. The charge q on the drop can then be computed by Eq. (55-3) of the preceding section. Substituting the weight w (or mg) of the oil drop for the force F in that equation gives

$$q = mg\, s/V \tag{56}$$

To find the number of electrons on a given drop, the drop must be observed over a considerable period of time. It will be found that the value of the charge on any drop changes abruptly from time to time. This is indicated by the fact that the potential difference required to support the drop must be suddenly changed from time to time. Between these sudden changes, it remains constant. Each constant value of V will indicate a different value of q according to Eq. (56). If we record the successive values of q for a given drop, we will find that the difference between any two values is never less than a certain observed amount e. We will also find that all the observed values of q are integral multiples of this quantity e. In other words, we will find that all values of q can be expressed in the form

$$q = ne$$

where n is always a small whole number and never a fraction. We therefore conclude that the observed value of e is the charge on one electron. In a typical experiment, n may be found to have values ranging from one to about ten.

The changes of q for a given drop of oil depend on its encounters with charged and uncharged molecules of gas. If the space between the horizontal plate is subjected to x-rays or radioactive radiations, the value of the charge will change more frequently as mentioned above.

57. The Acceleration of Free Charges in Electric Fields. A free, positively charged particle placed in an electric field will experience a force in the direction of the field. If it is at rest to begin with, it will start to move in the direction of the field, that is, from higher to lower potential. Thus its electric energy will decrease and its kinetic energy will increase as it moves with increasing velocity. According to the principle of conservation of energy, the gain in kinetic energy will be equal to the loss in electric energy. If the velocity increases from zero to a final value v while the charge q moves through a difference in potential V, then

$$Vq = (1/2)\, mv^2 \tag{57}$$

Here m is the mass of the particle, which is assumed to be constant. If the velocity of the particle changes enough so that the mass changes appreciably, an appropriate correction must be made in the value of m as explained in Sect. 14.

For charged particles of atomic or subatomic size, the mass is so small that gravitational forces on the particle are usually negligible compared to the electrical forces involved.

Equation (57) applies to the operation of x-ray tubes such as are commonly used in medical practice. A common type of x-ray tube consists of a metal filament F placed opposite a so-called

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1. $$w = 9\pi \sqrt{2v^3 n^3 / \rho g}$$

where v is the velocity of fall, n the viscosity of the air, ρ the density of the oil less the density of the air, and g the acceleration of gravity.

Brawny Jongs Brand

metal "target" A in a highly evacuated glass tube (Fig. 57-1). The filament is heated like the filament of an ordinary incandescent lamp bulb by a current from a battery or a small transformer T. Another high potential battery B is connected to maintain the target A at a high potential relative to the filament F. Electrons emitted from the hot filament are thus accelerated toward the positive target. When the electrons strike the target, the kinetic energy of some of them is completely converted into the desired x-rays which then radiate from the target. For other electrons, part or all of the kinetic energy may be converted into heat when they strike the target. This heat is a wasteful byproduct, and it may be great enough to melt the solid metal target if too much power is applied to the tube.

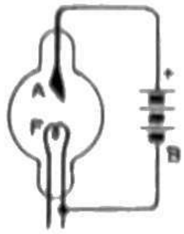

Fig. 57-1

Another example of the acceleration of electrons through a potential difference is found in television tubes. Parts of such a tube are illustrated in Fig. (57-2). The heated filament from which the electrons are emitted are shown at F. Next to the filament are two parallel plates with a hole in each one as shown in cross-section at A and B. The plate A is maintained at a positive potential relative to F. Electrons from F are thereby accelerated toward A. Some of them will pass on through the holes in A and B and emerge on the other side with a velocity that will depend on the potentials V_0 and V_1. Such an arrangement for projecting a stream of electrons is often called an electron gun.

In a television tube, the electrons that emerge from the electron gun afterwards pass near other conductors which are not shown in Fig. 57-2. These other conductors can be used to deflect the stream of electrons to any desired spot on the screen at the end of the tube. When they strike this screen, they expend their kinetic energy and produce light.

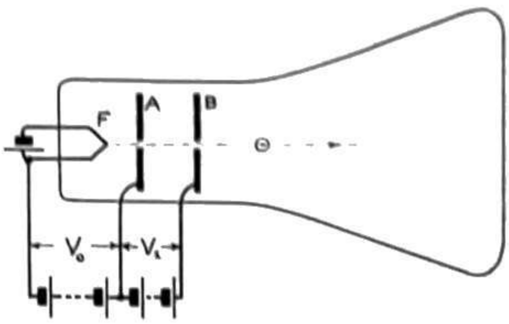

Fig. 57-2

In computing the kinetic energy of an electron moving through an electron gun, it may be assumed that an electron passing through a hole in a plate is so near to the plate that it must have reached the same potential that it would have reached if it actually hit the plate.

Chapter 6

ELECTROSTATIC INDUCTION

60. Induced Charges on Conductors. We have seen (Sect. 15) that any body of material contains charges of both signs, whether or not there is an excess of either kind. If an external charge Q is placed near an uncharged conductor, it will attract the unlike charges in the conductor and repel the like charges. The resulting displacement of movable charges in the conductor will thus leave certain areas of the conductor with an excess of one kind of charge. For example if a positive charge Q is placed near an uncharged conductor AB as shown in Fig. 60-1, the negative charges in the conductor will be attracted and the positive charges repelled. This will create an excess of negative charge in the nearer end A, and an excess of positive

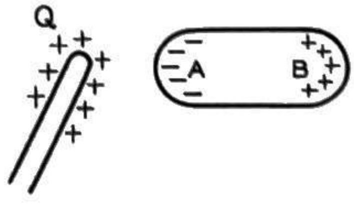

Fig. 60-1

charge in the more remote end B. Such charges produced by separation in the field of another charge are called induced charges. The charge that causes the separation is called the inducing charge. Since induced charges are produced by separation, two equal and opposite amounts must always appear at the same time.

The production of induced charges as considered above may be described in other words by using the concept of potential. From this point of view, the first effect of bringing the external charge Q near to the uncharged body AB is to create a difference in potential between the two ends A and B. Because A is closer to Q than B, it will be at a higher potential. Positive charges will then move toward the low potential end and raise its potential while negative charges will move toward the high potential end and lower its potential. This movement will continue until both ends of the conductor are again at the same potential. Thus while the approach of the positive charge Q raises the potential of the conductor, the separation of the induced charges keeps the increase in potential uniform for all parts of the conductor.

When equal and opposite charges are induced on a body, the unlike induced charge will always be nearer to the external charge Q. Hence the attraction of Q for the unlike induced charge will be greater than its repulsion for the like induced charge, and there will be a resultant attraction. Thus it is that a charged body can attract a neutral body by inducing charges on it.

Charges separated by induction will ordinarily move back together and neutralize each other when the inducing charge is removed. A permanent separation of the induced charges can be made by letting the repelled charges pass to another body through a temporary contact. Such contact must be made before the inducing charge is removed. One step of this process is illustrated in Fig. 60-2. Here a conductor C has just been touched to the body AB and the like induced charge has moved over to C. By breaking the contact between C and AB before Q is removed, the induced charges can be kept permanently separated even after Q is removed.

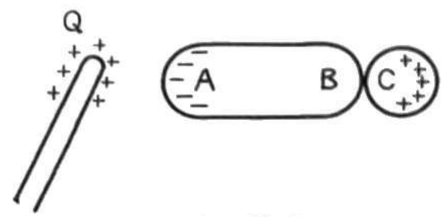

Fig. 60-2

61. Charging an Electroscope by Induction. An electroscope may be charged by the process of induction that was explained in the preceding section. The application of this process to an electroscope is interesting because the electroscope itself will indicate the changes in potential that occur. The steps of the process are shown in Fig. 61. In the first step a rod with a positive charge Q is brought near the top of the electroscope. Because of the nearness of this charge, the electroscope will be at a high potential. Also, induced charges will be separated on the electroscope as shown, and they will be distributed so that all parts of the electroscope shown are at the same high potential. In the second step the induced positive charge is conducted away from the electroscope by touching a finger to the top of the electroscope. Although there is an induced excess negative charge on the top end, there are also positive charges present in the material. The excess negative charges are held there by the attraction of the inducing charge, while the repelled positive charges move to the earth through the body of the person touching the electroscope. Positive charge will continue to flow until the potential of the electroscope is at the same zero potential as the earth. Note that the electroscope is then undeflected because it is left with just enough negative charge to neutralize the positive potential produced by the nearby inducing charge. (See Sect. 44)

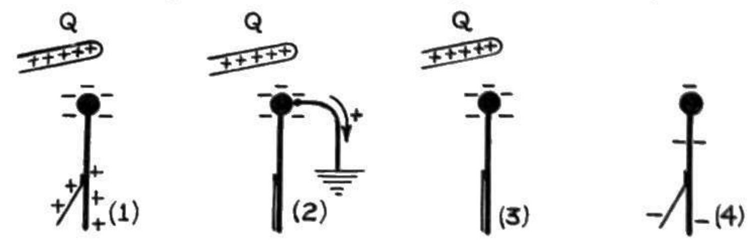

Fig. 61

In the third step, the finger is removed without changing the electrical conditions. In the fourth and final step, the inducing charge Q is removed. The electroscope is then left deflected

with the negative charge spread overthe whole electroscope to give a uniform potential. Induced changes of either sign can be obtained in this way. In general, the sign of the inducing charge must be apposite to the sign of the desired final charge.

62. The electrophorus is a simple device for obtaining a rather large charge by induction. It consists of a flat metal plate P, with an insulating handle H, and a sheet R of some insulator which has been charged by friction. R will have a negative charge if it is a sheet of hard rubber which has been charged by rubbing with cat's fur. When the uncharged metal plate is placed on top of the charged insulator, the charges in the metal plate will be separated by induction, as shown in Fig. 62-2. Although the metal plate rests on the charged insulator plate, the two plates actually touch at only a few high spots. Over most of the area, the two surfaces are close together without actually touching. Under these conditions, the amount of the negative charge that will be transferred directly to the metal plate at the points of contact will be so small that it may be disregarded in comparison to the charges which are induced on the metal plate. If the metal plate is temporarily touched with the finger while resting on the charged insulator, the negative charges will flow off through the finger. If the metal plate is then removed from the charged insulator, the metal plate will have a sizable positive charge. Note that in this procedure, the negative charge remains on the insulator, so that the process could be repeated over and over without appreciably using up the charge on the insulator. Thus, an unlimited amount of induced charge could be obtained from a small initial amount of inducing charge.

Fig. 62-1

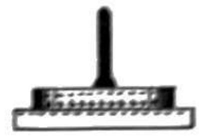
Fig. 62-2

The plate of an electrophorus can be charged by the process described above so that a spark will jump from the plate to a nearby neutral object over a distance more than a half an inch. The light and sound given out by the spark indicate that energy was stored in the plate. Since the original charge on the insulator is not used up, the energy cannot come from that source. Instead, the energy comes from the mechanical work which is done in pulling the induced positive charge on the metal plate away from the inducing negative charge on the insulator. Thus, the electrophorus is a simple generator for using up mechanical energy and converting it into electrical energy.

63. Electrostatic Generators. An electrostatic generator is a device that utilizes mechanical energy to concentrate a large electric charge at a high potential. It is therefore a device for converting mechanical energy into electrical energy. The electrical energy stored in the accumulated charges may be utilized by allowing them to all escape in one explosive discharge, or it may be utilized by allowing them to escape at a controlled rate. In the latter case the charge may be replaced by the generator as fast as they escape. Thus the generator may maintain a steady flow of charge and a continuous transformation of energy.

A very simple electrostatic generator capable of generating a theoretically unlimited potential can be had by using an electrophorus in connection with a large hollow metal "collector" mounted on an insulated support. This collector must be shaped so that the metal plate of the electrophorus can be inserted inside the collector before touching it. By repeatedly charging the metal plate and touching it to the inside of the collector, the entire charge on the plate will pass to the outside of the collector each time, no matter how highly the collector becomes charged. Thus, the process theoretically can be continued long enough to build up the potential to any desired value. A practical limit to the potential is set by the fact that it is impossible to perfectly insulate the collector. Hence, as the potential gets higher and higher, the charges begin to leak away as fast as they can be carried up to the collector.

The so-called static machine is a common type of electrostatic generator which employs rotating disks to carry charges from the point where they are induced to the point where they are accumulated. One of the features of a static machine is that it utilizes the discharge of

electricity from sharp points through the air instead of actual temporary contact for drawing off the like charge in the induction process. (See Sect. 54)

In the Van de Graaff type generator, electric charges are sprayed on to a moving rubber conveyor belt by sharp points from a comparatively low potential generator. These charges are then carried by the belt to the interior of a large, hollow, spherical conductor, which serves as a collector. Inside this sphere, the charges are removed from the belt by the action of sharp points, and left to go to the outside of the hollow collecting sphere. As explained above, the charges will pass to the outside of the collecting sphere regardless of how high the potential may already be. The inevitable leakage that limits the voltage can be decreased by mounting the spherical collector on a long insulating support. Generators have been built which are taller than a two story house and which are capable of generating millions of volts. The voltage-limiting leakage may also be retarded by surrounding the spherical collector and its insulating support with oil or compressed air. Smaller generators are often built with this protection.

64. Induced Charges on Insulators. A limited amount of charge can be induced on a body of insulating material by bringing an external charge near. Although the effect is not as marked as it is for a conducting body, it is enough to give a noticeable attraction between an external charge and an uncharged insulating body. This induced charge on insulating bodies is due to a slight displacement of the charges within each molecule of the body rather than to any motion of charges through the material.

There are certain kinds of wax in which the displacement of the charges in the molecules can be effectively frozen by letting the wax solidify in the field of the external inducing charge. The two ends of the piece will then retain their opposite charges after the inducing charge is removed, but the total charge on the piece will still be zero. Pieces of wax charged in this way are called electrets.

Chapter 7

CHEMICAL SOURCES OF POTENTIAL DIFFERENCE

70. Volta (1745-1827) discovered that a difference in potential is created between two pieces of different metals when the pieces are immersed together in an acid solution. For example, if a piece of copper and a piece of zinc are dipped in a beaker of dilute sulphuric acid, the copper will acquire a potential about one volt higher than that of the zinc. From an electric point of view, the chemical action that produces the potential difference can be referred to briefly in terms of an assumed "chemical force." In other words, the result is the same as if there was a chemical force to simply carry positive charges from the zinc to the copper through the solution. Such a combination is called a voltaic cell. The two conductors are referred to as the electrodes of the cell.

Voltaic cells are commonly used in practice as convenient sources of potential difference. The ordinary dry cell is a voltaic cell that has a moist paste instead of a liquid solution. One of its electrodes is a carbon post supported inside a zinc cup, with the zinc cup serving as the other electrode. The difference in potential furnished by such a cell is approximately 1.5 volts with the carbon post in the center positive. The maximum potential that can be generated by such a dry cell is the same regardless of its physical dimensions.

71. Electrochemical Series. The chemical action in a voltaic cell depends on the tendency of the electrodes to be dissolved in the solution. In this process, each atom that is dissolved leaves one or more of its electrons on the undissolved part of the electrode. Thus the tendency to dissolve is opposed by the attraction of a dissolving atom for its lost electrons. As a result,

the dissolving action will stop when the total negative charge left on the electrode exerts enough attraction to balance the tendency to dissolve. It follows that the maximum potential difference that will be built up between the solution and an electrode will depend on the tendency of that particular material to go into solution.

The tendency of a material to dissolve is often referred to as its solution pressure. The solution pressure of a material depends on its chemical properties, and hence each different chemical substance has a different solution pressure. It has been found that all metals can be arranged in a sequence according to the magnitude of their solution pressures. This sequence is referred to as the electrochemical series of metals. When two electrodes are immersed in the same solution, they both tend to acquire a negative potential relative to the solution. The one with the higher solution pressure will become more negative. Thus the potential difference between the electrodes of a voltaic cell depends on the two electrodes having different solution pressures. The farther apart the two materials are in the electrochemical series, the larger will be the difference in potential between the two electrodes.

72. The Electric Characteristics of Cells. The electric behavior of chemical cells will be considered in more detail later. For the present, a voltaic cell may be considered simply as a conducting path between two metal terminals, which allows charges to flow through in either direction as needed to maintain a fixed difference in potential between the terminals. It follows that any conductor connected to the positive terminal of a voltaic cell will be higher in potential than any conductor connected to the negative terminal by a fixed amount, provided a cell is not abused.

Voltaic cells are represented in drawings by the symbol shown in Fig. 72 where the long thin line represents the positive terminal as marked.

By connecting a number of cells in a series, with the negative terminal of one connected to the positive terminal of the preceding one, the successive steps in potential will add up so that the difference in potential between the extreme ends of the series will be the sum of the separate steps. Such a combination of cells is called a battery. Batteries of dry cells are commercially available giving up to 90 volts potential difference, and these batteries can themselves be combined in series to give still higher voltages.

Fig. 72

Chapter 8

ELECTRIC CAPACITORS

80. An electric capacitor, or condenser, consists essentially of a pair of insulated conductors such that a considerable amount of charge can be transported from one to the other for a given difference in potential between the two. The capacitors that are commonly used in radios and in other electrical devices generally consist of a pair of parallel metal plates. The plates may be self supporting with an air space between, or they may be made of thin metal foil with a sheet of paper or other insulating material between. In many capacitors, each conductor consists of a great number of sheets of metal fastened together and interleaved between the sheets of the other conductor, as illustrated in Fig. 80. The whole thing is often mounted inside a case, and the two conductors are provided with terminals for external connections.

The function of capacitors in electricity is analagous to the function of elastic springs in mechanics. A capacitor permits a displacement of electric charge from one plate to the other. This displacement builds up a potential difference with a restoring force on the charge back towards its original

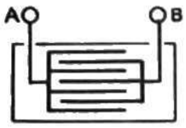

Fig. 80

position. Thus capacitors may serve as electric cushions, corresponding to mechanical shock absorbers. They may also serve to store energy as the main spring of a watch stores energy, and they may permit electric oscillations which are analogous to elastic vibrations.

A capacitor may be charged by connecting its terminals to some source of potential difference, such as a voltaic cell. This source will remove charge from one plate and deposit it on the other. If Q is the amount of charge transferred from the terminal B to the terminal A, then A will have a positive charge Q while B will be left with an equal negative charge Q. Except for slow leakage, a capacitor will remain charged after being disconnected from the source of potential difference. A capacitor may be discharged by placing a conducting path such as a piece of wire between its terminals. That will allow the charges to flow back to their original positions.

81. Capacitance. The capacitance of a capacitor is defined as the ratio of the transferred charge Q to the difference in potential V between the plates. A condenser with a large capacitance will permit a large charge to be displaced without building up much potential difference. It can be shown by theory and by experiment that the potential difference between the plates is proportional to the transferred charge. Thus the capacitance is a constant independent of the amount of charge involved. The definition of capacitance may be stated as an equation in the form

$$C = Q/V. \tag{81}$$

It follows from the definition that any unit of charge divided by any unit of potential difference could be used as a unit of capacitance. A coulomb per volt is used as a unit which is called a farad. Thus a capacitor would have one farad of capacitance if the transfer of one coulomb of charge produces a difference in potential of one volt. Most capacitors used in practice have capacitances which are only small fractions of a farad, so that the microfarad ($= 10^{-6}$ farad) and the micro-microfarad ($= 10^{-12}$ farad) are the most commonly used units of capacitance. To be consistent with the usage in connection with other units, the microfarad and the micro-microfarad should be abbreviated μfd and $\mu\mu$fd respectively, but the abbreviations mfd and mmfd, or MFd and MMFd are commonly used. In spite of this inconsistency, there is no confusion in practice because the millifarad and the megafarad are never used.

Note that the capacitance is not the maximum amount of charge which a capacitor can hold. There is a practical limit to the amount of charge a capacitor can hold due to the fact that the potential cannot be built up indefinitely without the charges breaking back through the insulator. In a capacitor insulated with air, this break would occur as a spark, or minature flash of lightning. Capacitors are frequently marked with the value of the capacitance and with a rated voltage. The rated voltage indicates the maximum voltage that can be safely applied, but the capacitor may be used at any lower voltage with the rated capacitance applying at any voltage used.

82. The capacitance of a parallel plate capacitor. If two parallel plates each of area A are charged with equal and opposite charges +Q and -Q respectively, the electric field between the plates when separated by empty space is given by the equation

$$\mathcal{E} = K_0\, 4\pi Q/A \tag{82-1}$$

as stated in Sect. 55. The difference in potential V between the plates will be given by

$$V = \mathcal{E}s \tag{82-2}$$

where s is the distance from one plate to the other, so that we may write

$$V = K_0\, 4\pi Qs/A \tag{82-3}$$

By rearranging the terms in Eq. (82-3) we see that the capacitance C of the two plates used as a capacitor may be written in the form

$$C = \frac{Q}{V} = \frac{1}{K_0\, 4\pi}\,\frac{A}{s} \tag{82-4}$$

By introducing a new quantity k_0 which is defined by writing $k_0 = 1/(4\pi K_0)$, we may write

$$C = k_0 \frac{A}{s} \tag{82-5}$$

Since $K_0 = 9 \times 10^9$ newt/(coul^2/m^2) as given in Sect. 17, the value of k_0 will be 8.85×10^{-12} coul^2/(m^2 newt). A meter-newton is one joule, and a joule/coul is one volt, so that k_0 may be written

$$k_0 = 8.85 \times 10^{-12} \text{ farad/meter} \tag{82-6}$$

83. Dielectrics. The values of K_0 and k_0 which have been given above apply where the space between the charges is empty space. If the space between the capacitor plates is filled with certain other insulators such as glass, oil or mica, the capacitance of the capacitor will be increased by a numerical factor k', which is called the specific inductive capacity of the insulator. Eq. (82-5) may be made to take this into account by writing

$$C = k' k_0 \frac{A}{s} \tag{83}$$

Typical approximate values of k' for a number of insulators are as follows:

air	1.00	paper	2.5
glass	8	mineral oil	2
mica	6	water	81

An insulator is often referred to as a dielectric, particularly when it is used between the plates of a capacitor, and the specific inductive capacity is sometimes called the dielectric constant.

The ability of a sheet of dielectric to increase the capacitance between two metal plates depends on the induced charges that appear on the dielectric when it is placed in an electric field. As explained in Sect. 64, the molecules of a dielectric may be distorted by an electric field so that one side of the molecule has a positive charge while the other has a negative charge. This is illustrated in Fig. 83, where each oval represents a molecule. The whole figure represents a cross section of a sheet of dielectric between the two metal plates A and B of a capacitor. The net effect of the distortion of the dielectric molecules is to give an induced positive charge on the left face of the insulating sheet, and an induced negative charge on the other face. The induced charge on either face of the dielectric is opposite in sign to the charge on the adjacent plate of the capacitor. This tends to neutralize the effect of the charges on the metal plates as far as creating a potential difference is concerned. Thus, when the dielectric sheet is in place, there will be a smaller potential difference for the same charge on the metal plates. It follows that the capacitance will be larger due to the dielectric, since the capacitance is the ratio of charge to potential difference.

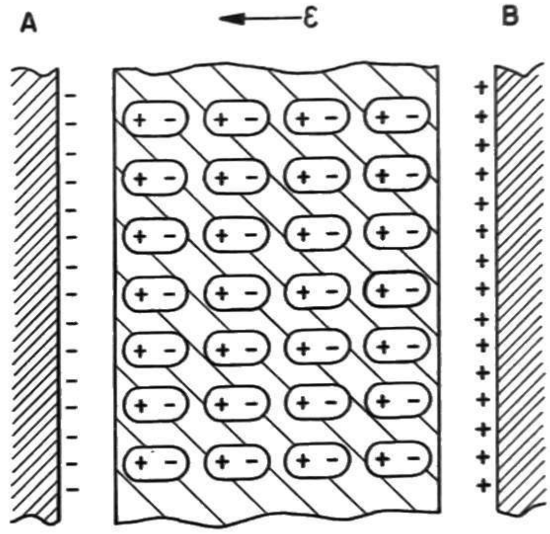

Fig. 83

When a condenser with a dielectric is discharged, the induced charges on the surface of the dielectric disappear into the dielectric as the charges in the distorted molecules return to their normal positions. If a dielectric is subjected to an alternating electric field, the consequent distortion of the molecules first in one direction and then in the other generates heat in the interior of the dielectric. Such dielectric heating has found commercial application in processes like

bonding plywood together or roasting food. In such processes, the production of heat in the body of the material is desirable to give a uniform temperature throughout.

84. Electric Permittivity. The constant k_0 which appears in Eq. (82-5) above is known as the permittivity of empty space. The product $k'k_0$ for any material having a specific inductive capacity k' is known as the permittivity of that material.

85. The Energy of a Charged Capacitor. The energy stored in a charged capacitor is taken to be the energy that can be gotten out of it by completely discharging it. That will be equal to the work which the total charge Q on the capacitor can do as it moves from the high potential terminal to the low potential terminal through the discharging path. The energy will therefore be $Q\bar{V}$ where $\bar{V}$ is the average potential for the discharge.

When the capacitor is discharged, the first charges flow from high to low potential while the potential still has its initial high value. The last charges flow across when the potential has dropped practically to zero. Since the potential at any instant is proportional to the charge, the average potential $\bar{V}$ during the discharge will be one half of the original potential V. Hence a capacitor charged with a charge Q to a potential V will have an energy W given by

$$W = Q\bar{V} = Q\,V/2 \tag{85}$$

86. Parallel Combination of Capacitors. Two or more capacitors connected in parallel between one pair of terminals M and N must thereby always be charged to the same potential difference V. The total charge Q that is transported from one terminal to the other will be the sum of the charges Q_1, Q_2, etc., on the separate condensers. The capacitance C of the combination between the two terminals M and N will thus be given by

$$C = \frac{Q}{V} = \frac{Q_1 + Q_2 + Q_3 + \dots}{V} = C_1 + C_2 + C_3 + \dots \tag{86}$$

Chapter 9

ELECTRIC CURRENTS IN CONDUCTING PATHS

90. Electric charges will move through a body of conducting material whenever there is a difference in potential to make them move. In general, the flow of charge in a large body is a three-dimensional phenomena similar to the flow of particles in a tub of water that has been stirred. In this book we will be chiefly concerned with cases where the flow of electricity is confined to narrow conductors like copper wires. This type of flow corresponds to the flow of water in a pipe.

A conductor that confines the flow of electricity along one direction will be referred to as a path. One of the characteristics of such a path is that all the charges which enter one end must flow through the entire length and come out at the other end.

The flow of electricity through a conducting path is of interest not only because the electricity is moved from one place to another but also because of the effects produced while it is moving. For example, the flow of current in a wire may generate heat or produce a mechanical force on the wire. The flow through a liquid conductor may produce chemical effects such as the electroplating of metals.

The filament of an incandescent electric lamp is a good example of an electric path used in practice. The filament is usually a piece of tungsten wire with the ends fastened to external connections at the base of the bulb. In operation, electric charges flow in to the filament at one end and out at the other, producing heat as they pass along the wire.

91. Electric Current. In all the effects that we observe due to the motion of electric charges along a path, the rate of flow is an important factor in determining the magnitude of the effect. The amount of charge that passes a given point in a path per unit time is referred to as the electric current I in the path. Thus if Q is the amount of charge that passes in a time t, the average current I will be given by the equation

$$I = Q/t \tag{91}$$

It follows from the definition of current that any unit of charge divided by any unit of time may serve as a unit of current. Thus if a charge of 80 coulombs passes a point in 10 seconds, the average current during that time is 8 coulombs per second. A coulomb per second is more briefly referred to as an ampere. Other units of current often used include the following:

microampere	(= 10^{-6} amp)
milliampere	(= 10^{-3} amp)
abampere	(= 10 amp)

Following common usage, the positive direction of a current is taken to be the direction in which positive charges would have to flow to produce the observed effect, regardless of which kind of charge is actually moving. (See Sect. 24)

92. The Maintenance of an Electric Current in a Path. In order to maintain a steady flow of current through a conducting path, it is necessary to maintain a potential difference between the ends of the path. For example, suppose we wish to maintain a current I flowing through the filament of a lamp bulb from A to B as shown in Fig. 92. To keep A at a high potential and B at a low potential, we must remove charges from B as fast as they arrive and place them back on the other end at A. In practice, this can be done by connecting the two ends of the path to some source of electric energy C. This source may be a voltaic cell (Sect. 70) or it may be a generator in a distant power house to which connection is made through a wall outlet. When connected in this way, the path AB and the source C form a closed circuit, and the source of power keeps the charges circulating around in the circuit as indicated by the arrow tips.

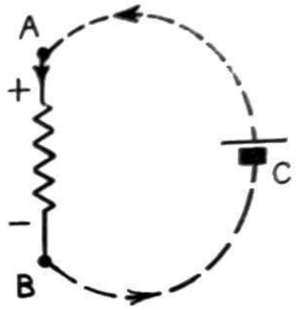

Fig. 92

The direction of the current along a simple conducting path will always be from high to low potential. That is the direction of the electrostatic force associated with the difference in potential. In drawings, plus and minus signs are often used in pairs to indicate the difference in potential between the two ends of a current carrying path. The plus indicates the end of higher potential. This convention is illustrated by the signs placed at the ends of the path AB in Fig. 92. Signs used in pairs in this way have significance with regard to the included path only, and neither one has any meaning with regard to any sign in another part of the circuit.

93. Ohm's Law and Electric Resistance. In the preceding section it was pointed out that the charges in an electric current will not continue to flow past the obstructing atoms unless a difference in potential V is maintained between the ends of the path. For a given path it is found that the difference in potential required to maintain a desired current is proportional to the current. This proportional relationship is known as Ohm's Law. Ohm's law may be expressed mathematically by writing

$$\frac{V}{I} = R. \tag{93}$$

Here R is a constant factor of proportionality with reference to variations in V and I. The fixed ratio of V/I which holds for a given path is called the resistance of the path. Thus Eq. (93) may

be regarded both as a statement of Ohm's law and as a definition of the resistance R. The application of the name resistance to this ratio is appropriate, because if it takes more potential difference to force a given current through a path, the path may well be said to offer more resistance to the flow. The resistance of a given path such as a piece of wire depends on the kind of material, the cross-sectional area, the length, and the temperature of the wire. The influence of these various factors will be discussed in more detail later.

Any unit of potential divided by any unit of current may be used as a unit of resistance. The so-called practical unit of resistance is the ohm, which is defined as a volt per ampere. Thus if a difference in potential of 120 volts will drive a current of 2 amperes through the filament of a lamp bulb, the resistance of that path will be

$$R = \frac{120 \text{ volts}}{2 \text{ amp}} = 60 \frac{\text{volt}}{\text{amp}} = 60 \text{ ohms}$$

The megohm (10^6 ohms) and the microohm (10^{-6} ohms) are other units used to measure very large and very small resistances respectively.

Equation 93 can obviously be written in the form $V = IR$. A potential difference associated with a current flowing through a resistance in accordance with this equation is often referred to as an "IR drop." In other words, the potential drops by an amount IR as we move with the current from the high to the low potential end.

Ohm's law may be demonstrated in graphical form by plotting the values of potential difference V against the corresponding values of the current I for a given path. According to Ohm's law, the resulting graph must be a straight line through the origin. The constant slope of this line will be equal to the resistance of the path.

94. The Measurement of Electric Current by Moving-coil Current Metus. An electric current can be measured by sending it through a coil of wire properly mounted between the poles of a magnet. One arrangement which can be used for this purpose is shown in Fig. 94. In practice, the flat rectangular coil generally consists of a number of turns of wire wound on a lightweight frame. The coil is suspended by an elastic metal strip which holds the coil to its zero position when no current is flowing. This strip also serves to conduct the current from one binding post to the coil. The flexible spiral below the coil conducts the current from the coil to the other binding post. The cylindrical piece of iron at the center of the coil concentrates the influence of the magnet without interfering with the rotary motion of the coil.

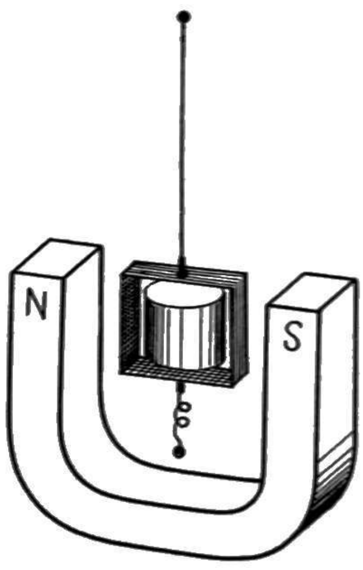

Fig. 94

As will be explained later in Chap. 20, a current flowing through the coil of Fig. 94 will make the coil rotate in a horizontal plane against the restoring torque of the elastic suspension. For a given current, the coil will rotate and come to rest in a position where the torque from the current is balanced by the restoring torque of the elastic suspension. The greater the current, the greater will be the angular deflection, and hence the deflection gives a measure of the current. The angular displacement of the coil can be observed by attaching a pointer to the coil and providing a graduated scale at the end of the pointer. Some meters use a weightless beam of light as a pointer by reflecting the beam from a small mirror attached to the coil.

The arrangement shown in Fig. 94 is known as a suspended moving-coil, or D'Arsonval,

galvanometer. Commercial meters of this type are capable of measuring currents as small as 10^{-9} amperes.

95. Portable Current Meters. The suspended coil type of galvanometer described in the preceding section is a rather delicate instrument and must be kept in an upright position. To give a more rugged instrument that can be carried about and used in any position, the moving-coil can be pivoted between two jewel bearings placed respectively at the top and bottom of the coil frame. Compact instruments constructed in this way are commercially available. In these instruments, the elastic support required to hold the coil to its zero position is usually furnished by flat spiral springs, one attached at each bearing. These spirals also serve as flexible connectors to complete the path through the moving coil from one stationary binding post to the other. The coils in these meters are generally provided with rigid pointers.

Pivoted-coil meters can be made to measure large or small currents depending on the number of turns in the coil, the stiffness of the elastic springs and other features of the design. Since the pivots inevitably introduce some friction, a pivoted coil meter can never be made as sensitive as the suspended coil type.

The term galvanometer which is generally applied to a suspended coil meter is also applied to a pivoted-coil instrument if the meter is relatively sensitive, and if the scale is marked in arbitrary divisions. However the scales of pivoted-coil meters are generally calibrated to indicate the current directly in units of current. These meters are then referred to as ammeters, milliammeters, or microammeters, depending on the unit in which the scale is calibrated.

96. Galvanometer Sensitivity and Resistance. A current meter has two important characteristics which determine its suitability for a given use. These are its sensitivity and its resistance. The sensitivity of a galvanometer is the amount of current indicated per smallest division graduated on the scale. For example, if a current of 40 microamperes deflects the pointer of a galvanometer 5 divisions the sensitivity is 8×10^{-6} amperes per division. The resistance of a galvanometer is simply the resistance of the conducting path through the coil from one terminal to the other.

97. Moving-coil Voltmeters. Since the path through a galvanometer or milliammeter has a definite resistance, the current through the meter is proportional to the difference in potential between its terminals. Thus the deflection which is proportional to the current will also be proportional to the difference in potential between its terminals. This means that the same pointer which indicates current can also be made to indicate the potential difference between the meter terminals, and therefore between any two points connected to these terminals. The meter is then a voltmeter. The characteristics of a meter which makes it particularly useful as a voltmeter or an ammeter will be discussed in detail later. (See Sect. 116)

Chapter 10

ELECTRIC CURRENTS IN CONDUCTING PATHS (continued)

100. Energy and Power Expended in a Resistive Path. When electric charges pass through a simple conducting path having resistance, they come out of the low potential end with less electrostatic energy than they had at the high potential end. The loss in energy by the charges is due to their collisions with the fixed atoms of the conductor, and the lost energy is thereby transformed into heat energy. The generation of heat is an unavoidable result of a current flowing through a resistance, just as the generation of heat is an unavoidable result of motion where there is mechanical friction.

If a total charge Q passes through a conducting path in a given time t, and if the difference in

potential between the ends of the path is V, the total expenditure of electric energy W will be VQ. If the current I is constant, Q = It so that we may write

$$W = VIt \tag{100-1}$$

Equation (100-1), written in the form

$$V = W/It \tag{100-2}$$

suggests that the difference in potential between the ends of a simple conducting path can be measured by measuring the amount of heat generated in the path when a measured current I flows for a measured time t. This method of measurement is based directly on the definition of potential difference as energy per unit charge, and may thus be considered to be a very direct measurement. The method is suitable for a laboratory exercise involving measuring techniques of various kinds, but because of its inconvenience it is seldom used in general practice.

Since power is the rate with which energy is expended, it follows from Eq. (100-1) above that

$$\text{Power } (P) = \frac{VIt}{t} = VI \tag{100-3}$$

Electrical power is usually measured in watts or kilowatts. The watt is a joule per second, and a kilowatt is equal to one thousand watts.

The use of units in Eq. (100-3) may be illustrated by finding the power expended in an electric stove which takes 11 amperes of current from a power line furnishing a difference in potential of 120 volts. Substitution in Eq. (100-3), gives

$$\begin{aligned}\text{Power} &= 120 \text{ volts} \times 11 \text{ amp} \\ &= 1320 \text{ volt amp} \\ &= 1320 \frac{\text{joule}}{\text{coul}} \frac{\text{coul}}{\text{sec}} \\ &= 1320 \frac{\text{joule}}{\text{sec}} \quad \text{or} \quad 1320 \text{ watts}\end{aligned}$$

A kilowatt-hour is a unit of energy based on the kilowatt of power. It is the amount of energy expended in one hour when the rate of expenditure is one kilowatt. In general, energy is equal to the product of power and time, so that any unit of power times a unit of time could be used as a unit of energy. Thus a watt-second is a joule.

101. Equations for Energy and Power in Terms of Resistance. The fundamental equations used to define power, P, potential difference V, current I and resistance R are

$$P = W/t \tag{101-1}$$
$$V = W/Q \tag{101-2}$$
$$I = Q/t \tag{101-3}$$
$$R = V/I \tag{101-4}$$

where W is the energy, Q the charge and t the time. These equations may be combined by simple algebra to derive other equations which are convenient for certain routine calculations. Some of these other equations commonly used are

$$W = I^2Rt \tag{101-5}$$
$$W = (V^2/R)t \tag{101-6}$$
$$P = I^2R \tag{101-7}$$
$$P = V^2/R \tag{101-8}$$

The student may easily verify the derivation of these equations from the four given above.

102. Applications of Resistive Heaters. A large number of electrical devices such as incandescent lamp bulbs, toasters, stoves, etc., are used to convert electrical energy into heat energy by merely allowing a current to flow through a resistance. From an electrical point of view, such devices consist essentially of a conducting path having a certain resistance. In designing any such device, the resistance is chosen so that when the device is connected to the potential difference for which it is intended, enough current will flow to give the desired power output. Such devices are usually marked both with the voltage for which they were designed, and with the power output that will result when that voltage is used. From these data, the corresponding rated values of the current and resistance may be compouted.

It should be noted that if the voltage applied to a resistive heating device is different from the rated voltage, then the actual current and actual power output will be different from the rated values. If the applied voltage is less than the rated value, the current and power will not be enough to bring the device up to its proper operating temperature. If the applied voltage is more than the rated value, the device may get too hot and burn out quickly.

The resistance of a heating device as computed from its rated power and voltage will be the value which holds when the device is at the proper operating temperature. For the tungsten filament of a lamp bulb, the resistance increases with the temperature so that the resistance at the operating temperature will be several times as great as the resistance at room temperature. For the alloys which are commonly used in the resistance elements of many electrical heating devices, the resistance does not change much between room temperature and operation temperature.

103. Conduction Paths through Empty Space. Many types of electronic tubes constitute conducting paths where the current is a stream of electrons moving through empty space. For example, the flow of electrons through an x-ray tube as described in Sect. 57 is externally equivalent to a conventional current I flowing through the tube in the opposite direction. Ohm's law will not in general hold for such a path, and the resistance will be different for different values of I. The equation $W = VIt$ however, is a general equation which always gives the energy expended in a path. In the case of the x-ray tube, this expended energy goes first to accelerate the electrons through the empty space, and give them kinetic energy. This kinetic energy then appears at the target of the tube when the electrons strike it. Some of the energy is converted into the desired x-rays, but most of it is converted into heat.

The production of heat in the target of an x-ray tube is an undesirable but unavoidable waste of energy. The same thing is true of many other types of electronic tubes. On the other hand electronic bombardment has been used to advantage in certain applications where it is desired to heat small bodies inclosed in an evacuated tube. Temperatures high enough to melt any of the metals can be obtained in this way.

104. The Conservation of Energy in Electric Circuits. The principle of the conservation of energy applies to electric circuits as elsewhere. In the circuit of Fig. 92, for example, the source of electric energy does work in carrying the charges from low to high potential on the one side of the circuit. This energy is lost by the charges and converted into heat as they flow down on the other side. According to the principle of conservation of energy, the net energy supplied per unit charge by the source of power in one part of the circuit must exactly equal the energy per unit charge lost as heat in the rest of the circuit.

Since potential difference is energy per unit charge, the principle of conservation of energy applies indirectly to potential differences. This fact has already been implicitly recognized in some of our discussions involving potential differences. In general, it tells us that the total potential difference encountered in a number of steps is the algebraic sum of the separate steps of potential difference. For example, consider three paths connected together end to end as shown in Fig. 104-1. If V_1, V_2 and V_3 are the net gains in potential encountered by a charge in

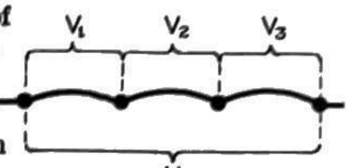

Fig. 104-1

passing through the respective paths, then the net gain V in passing from A to B will be given by

$$V = V_1 + V_2 + V_3 \tag{104-1}$$

A drop in potential encountered along any one of these paths may be added in algebraically as a negative gain.

The principle of conservation of energy also indirectly requires that if several different paths pass between the same two end points, they must all have the same difference in potential between their ends. For example, if V_1, is the gain in potential encountered in passing from C to D along the first path, V_2 the gain from C to D along the second path, and so on, then

$$V_1 = V_2 = V_3 \tag{104-2}$$

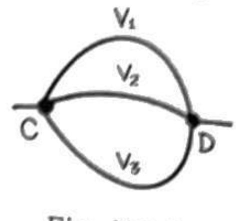

Fig. 104-2

105. Kirchhoff's Law for Currents. Electric currents in conducting paths behave as if electric charge can not be created or destroyed. In other words, whenever a steady potential drives a steady current through a path, the current which comes out at one end must be equal to the current going in at the other end. If several paths meet at a common junction point, the sum of all the currents that flow in to the point must be equal to the sum of all the currents that flow away from the point. This last statement is known as Kirchhoff's First Law for Electric Networks. Applied to a junction between two paths as shown in Fig. 105, it means that $I_1 = I_2$.

Fig. 105

Chapter 11

COMBINATIONS OF RESISTIVE PATHS

110. Equivalent Resistance of a Combination of Resistances. In practice it often happens that a number of resistive paths are connected together, and connected to the same source of potential. For example all the electric devices in a residence are usually connected together and all of them are connected to the same incoming power lines. We shall now consider the properties of such combinations.

Let A and B represent the two points on a given combination of resistive paths where the outside source of potential difference V is applied. In general, there will be a certain net current I flowing in to the combination through the high potential lead. The same current will flow out of the combination and back to the source through the low potential lead. The ratio of the difference in potential V applied between A and B to the net current I flowing through the combination from A to B is called the equivalent resistance of the combination. In other words, the equivalent resistance of a network is the resistance of a single path that could be connected to the source of potential difference to take the same current as the network takes. It follows that we would not be able to tell the difference between a combination of resistive paths, and a single equivalent path if both were inclosed in similar boxes with only the terminals exposed.

111. Series Combination of Resistances. If a number of resistive paths are connected together end to end so that the current which comes out of one path must all pass through the next, the paths are said to be connected in series. Such a combination is illustrated in Fig. 111. From what has just been said we may write

$$I = I_1 = I_2 = I_3 = \dots \tag{111-1}$$

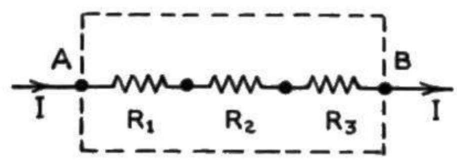

Fig. 111

where I is the current through the combination I_1 the current through the first resistance, and so on. If V_1, V_2, V_3 represent the respective potential drops in the several paths, then the total drop V across the combination will be given (See Sect. 104) by

$$V = V_1 + V_2 + V_3 \tag{111-2}$$

Since Ohm's law holds for each path separately

$$V_1 = I_1R_1, \quad V_2 = I_2R_2, \quad \text{and} \quad V_3 = I_3R_3 \tag{111-3}$$

The ratio of V to I is the equivalent resistance R of the combination according to the definition in the preceding section. Hence we may write

$$R = \frac{V}{I}. \tag{111-4}$$

Substituting for V from Eq. (111-2) gives

$$R = \frac{V_1 + V_2 + V_3}{I} \tag{111-5}$$

Using $V_1 = I_1R_1$ etc., we can then write

$$R = \frac{I_1R_1 + I_2R_2 + I_3R_3}{I} \tag{111-6}$$

Since the current in any one resistance is the same as the current I through the combination, the I's will cancel, giving

$$R = R_1 + R_2 + R_3 \tag{111-7}$$

It follows from Eq. (111-7) that the equivalent resistance of any series combination of paths will always be larger than the largest one of the single resistances.

For any two resistances in a series combination, the voltage drops across the respective resistances will be proportional to the resistances. This is true because $I_1 = I_2$, and hence $V_1/R_1 = V_2/R_2$ or

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} \tag{111-8}$$

112. Parallel Combinations of Resistances. If a number of resistive paths having resistances R_1, R_2 and R_3 are connected between two given points so that the same difference in potential V is applied to all the paths as shown in Fig. 112 the paths are said to be connected in parallel. It follows from Kirchhoff's law for currents in Sect. 105 that the current I coming up to the combination from the outside at the junction point C will divide among the several paths of the combination so that

$$I = I_1 + I_2 + I_3 + \dots \tag{112-1}$$

We also know from Sect. 104 that

$$V = V_1 = V_2 = V_3 \tag{112-2}$$

where V is the potential difference between C and D. Ohm's law holds for each path separately, so that

$$I_1 = \frac{V_1}{R_1}, \quad I_2 = \frac{V_2}{R_2} \quad \text{and} \quad I_3 = \frac{V_3}{R_3} \tag{112-3}$$

The equivalent resistance R of the combination between C and D is given by definition as

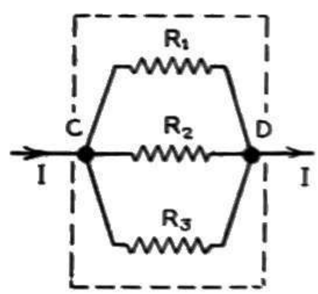

Fig. 112

$$R = V/I \tag{112-4}$$

To obtain an expression for R in terms of R_1, R_2, and R_3, we can substitute from Eq. (112-3) in Eq. (112-1) to give

$$\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots \tag{112-5}$$

Since $V = V_1 = V_2 = V_3$, all V's will cancel giving

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \tag{112-6}$$

It follows from Eq. (112-6) that the equivalent resistance of any parallel combination of paths will always be less than the smallest single resistance involved. This appears reasonable enough if we recall that a resistance is a conducting path, so that any added path will carry added current between two given points.

For any two of a number of resistances in parallel, the currents through these resistors will be inversely proportional to the resistances. This follows because $V_1 = V_2$, whence $I_1R_1 = I_2R_2$

or
$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \tag{112-7}$$

113. Computation of the Equivalent Resistance of any Combination. According to its definition, the equivalent resistance of any combination of paths from one point to another can be measured by applying a measured difference in potential V to these two points and noting the current I that flows in to the combination at one point and out of the combination at the other point. The equivalent resistance can also be found by computation for any type of network if the resistances of all the separate paths are known. In many common networks there will be found local groups of two or more resistances combined in series or in parallel. Any one of these groups can be replaced by a single resistance, computed according to the series or parallel equation. A somewhat simplified network can then be drawn in which each of these local groups is replaced by a single equivalent resistance. Successive applications of this simplifying process will eventually reduce the original network to one equivalent resistance between the two points of external contact. Other types of networks may require a more complicated method of solution.

Students should note that the terms series and parallel apply to the electrical connections rather than to the positions of resistances as represented on drawings. Thus in Fig. 113 the resistances are both connected in parallel "between" the points A and B.

It should also be noted that a given network of resistive paths may have different values for its equivalent resistance, depending on what two points of the network are to be connected to the external source of potential difference.

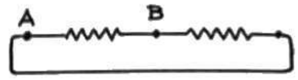

Fig. 113

114. Potential Dividers. A potential divider is a device for furnishing a variable potential difference by tapping off a portion of a given potential drop. It consists essentially of a series of resistors AB through which a current I is made to flow by some source of power, and a tap T which may be connected across any desired portion of AB to give a potential difference V. If no appreciable current is drained off through the tap T, the current through all parts of R_0 will be the same, and the total drop through R_0 will be IR_0. The value of V will be IR, so that V may be made to have any value between zero and IR_0 by moving the tap T. The resistance between A and B may be a bare wire along which the tap T can slide from one end to the other, in which case it is commonly referred to as a slide-wire.

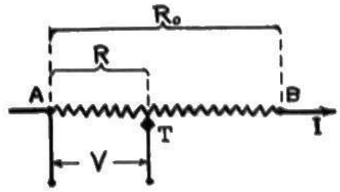

Fig. 114

Small potential dividers are widely used in radio applications, and are commercially available under the name of potentiometers. Except for this engineering usage, the name potentiometer is usually reserved to apply to a more elaborate piece of apparatus used for measuring potential differences. (See Sect. 172)

115. Ammeter Shunts. If we wish to measure a current in a given path by inserting an ammeter in series with the path, the resistance of the ammeter must be relatively small in order not to affect the current we wish to measure. In practice, ammeters with suitably low resistance are generally made by using a sensitive galvanometer in parallel with a low resistance shunt. The current through the galvanometer will then be a small but definite fraction of the current flowing through the combination, and the scale can be marked to indicate the current flowing through the combination rather than the current flowing through the galvanometer only.

Consider an ammeter made by shunting a galvanometer as shown in Fig. 115. Let A and B represent the external terminals of the ammeter, while I represents the current being measured. Inside the ammeter, the current is separated into two parts, with a current I_G passing through the galvanometer coil and a current I_s flowing through the shunt. The drop in potential from A to B through the resistance R_G of the galvanometer will be the same as the drop through the resistance R_s of the shunt, so that we may write

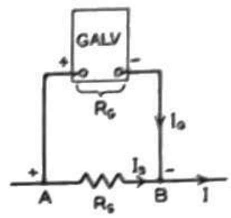

Fig. 115

$$I_s R_s = I_G R_G \tag{115-1}$$

or

$$\frac{I_s}{I_G} = \frac{R_G}{R_s} \tag{115-2}$$

Now since $I_s = (I - I\ \ )$ we can substitute in Eq. (115-2) to give

$$\frac{I - I_G}{I_G} = \frac{R_G}{R_s} \tag{115-3}$$

For ammeters and milliammeters constructed by shunting sensitive galvanometers, I_G is very much less than I and an approximate relation

$$\frac{I}{I_G} \doteq \frac{R_G}{R_s} \tag{115-4}$$

can be used.

Any ammeter behaves as a single conducting path between its two terminals, and it is impossible to tell from the outside whether the meter has an internal shunt or not. If an ammeter does have an internal shunt, the observable resistance of the meter as a whole will be the equivalent parallel resistance of the two internal paths. The observable sensitivity of the meter as a whole will be the amount of total current per scale division.

Any current meter can be made into a less sensitive current meter by an external shunt across its terminals. One sensitive meter and a number of relatively inexpensive external shunts may thus serve to measure a wide range of current values. External shunts are ordinarily designed so that the total current flowing through the shunt and meter in parallel is some simple multiple of the current through the meter itself. The theory involved in the above equations for a galvanometer with a shunt applies equally well to an ammeter with an external shunt.

116. Voltmeter Multipliers. Since the difference in potential between the terminals of a moving-coil current meter is proportional to the current, the scale can be marked to indicate the difference in potential in volts between the terminals, as explained in Sect. 97. An ordinary current meter may not have very desirable characteristics when used as a voltmeter. In order for a voltmeter to measure a potential difference without changing the potential to be measured, it

must be designed to take a very small current from the source being measured. Thus a moving-coil meter in general must have a very high resistance to serve as a voltmeter, and it must deflect with a very small current.

In practice, sensitive moving-coil meters having high resistance can be calibrated directly as voltmeters, but most voltmeters are made using a galvanometer with an added resistance connected in series as a so-called multiplier. A multiplier is always used where large voltages are to be measured, and it has a number of advantages for meters reading lower voltages. For example, a manufacturer may utilize the methods of mass production to make a single type of galvanometer coil, and then provide meters of different ranges by simply using different series resistors.

The exact amount of added resistance required for a given voltmeter can be easily computed. Let V be the differencein potential which is to be measured between the two external terminals of the voltmeter. Let I_o be the current that must flow through the galvanometer to give the desired deflection, as determined by the sensitivity of the meter. Let R_o represent the resistance of the galvanometer. The added resistance R must then have a value such that

$$I_o = \frac{V}{R_v} \tag{116-1}$$

where R_v is the sum of R_o and R. The resistance R_v of this series combination will then be the resistance of the voltmeter between its external terminals. From an outside point of view, such a voltmeter will appear as a single path having a resistance R_v.

Any voltmeter can be converted into a voltmeter of higher range by the addition of an external series resistance, or multiplier. The theory needed to compute the value of an external multiplier resistance is the same as the theory involved in Eq. (116-1) above.

The extent to which a voltmeter meets the requirement of taking a small current is generally specified in terms of the reciprocal of the current required for full scale deflection. This reciprocal may be found by dividing the resistance of the meter by the voltage required for full scale reading, and it is commonly expressed in ohms per volt. Thus a meter with 1000 ohms per volt takes a current of 1/1000 ampere for full scale deflection. Such a voltmeter with a full scale reading of 3 volts would accordingly have a resistance of 3000 ohms.

117. The Measurement of Resistances with Moving-coil Metus. The most direct way to measure a resistance R according to its definition is to measure the voltage V required to send a known current I through the resistance to be measured. Moving-coil meters can be conveniently used to measure V and I, if we are careful to allow for one difficulty which arises in practice. In practice, we must either connect the ammeter in series with R and measure the potential difference across the combination, or we must connect the voltmeter in parallel with R and measure the current through the combination. With the first connection, the ratio of the voltmeter reading to the ammeter reading will be the equivalent resistance of the series combination. With the second combination, the ratio of the readings will be the equivalent resistance of the parallel combination. Either of these equivalent resistances will approximate the unknown resistance if the ammeter resistance is relatively small and if the voltmeter resistance is relatively large.

118. Comparison of Resistances by the Ratio of Potential Drops. An unknown resistance X can be measured in terms of a known standard resistance R by connecting the two resistances in series so that the same current flows through both of them. The respective potential drops V_x and V_R will then be

$$V_x = IX \text{ and } V_R = IR.$$

Hence

$$X/R = V_x/V_R . \tag{118}$$

By measuring the potential drops with a voltmeter, the ratio of the drops can be used to give the ratio of the unknown to the known resistance.

Only one meter with its associated possible error is involved in this method, so that it will in general be a more accurate method than the one described in the preceding section.

119. Comparison of Resistances by Wheatstone's Bridge. Wheatstone's bridge is a combination of resistances as shown in Fig. 119. This arrangement is used to compare the potential drop across a known and unknown resistance more accurately than can be done by using a voltmeter as described in the preceding section. In the diagrams shown, X is the unknown resistance to be measured, and R is the known resistance to which it is being compared. R_1 and R_2 are an additional pair of known resistances. A battery C is used to send a current through the network from A to B, and a galvanometer G is connected between J and K. Different values of the known resistances are chosen by trial until no current flows through the galvanometer, and the bridge is then said to be balanced.

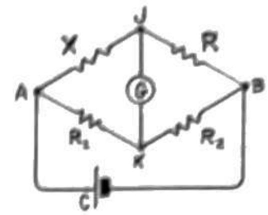

Fig. 119

If no current flows through the galvanometer, the difference in potential between its terminals must be zero. In other words, the potential at J must be equal to that at K. Hence the potential drop V_x across X must be equal to the potential drop V_1 across R_1. Similarly V_R must be equal to V_2, using a corresponding notation. It follows then that

$$\frac{V_x}{V_R} = \frac{V_1}{V_2} \tag{119-1}$$

Since no current flows through the galvanometer, the current I_x through X must be the same as the current through R, so that

$$\frac{V_x}{V_R} = \frac{X}{R} \tag{119-2}$$

Also the current I_1 through R_1 must be the same as the current through R_2, so that

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} \tag{119-3}$$

By substituting from Eqs. (119-2) and (119-3) in Eq. (119-1) we then obtain

$$\frac{X}{R} = \frac{R_1}{R_2} \tag{119-4}$$

One advantage of measuring resistance by Wheatstone's bridge is that the accuracy of the result is not affected by errors in the meter, since the meter is merely used to indicate the absence of a current through that path. The only accurate pieces of apparatus needed are the known resistances. Compared to ammeters and voltmeters, an accurate, known resistance is relatively inexpensive and easy to maintain, because it consists of nothing but a piece of wire with convenient terminals.

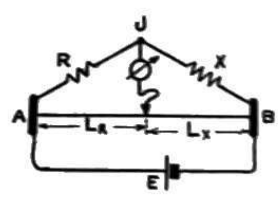

Fig. 120

For convenience in adjusting the known resistances in a bridge, at least one of them may be a dial type in which the resistance can be varied by turning a knob. In such resistors, the value of the resistance for any setting of the knob may be indicated by a pointer attached to the knob.

In a Wheatstone's bridge as described above, the resistances R_1 and R_2 can be furnished by two portions of a uniform "slide wire" AB as indicated in Fig. 120 . The slide wire, unlike other connecting wires, has a considerable uniform resistance per unit

length. The bridge is balanced by finding the point along the wire where the contact K must be to give no current in the galvanometer. The ratio of the resistances R_1 and R_2 will then be the same as the ratio of the corresponding lengths of wire so that

$$\frac{L_1}{L_2} = \frac{X}{R} \tag{119-5}$$

Chapter 12

THE RESISTIVE PROPERTIES OF MATERIALS

120. The Resistivity of Materials. When a steady current flows through a relatively long uniform metal wire or strip, the moving charges will be spread almost uniformly over the cross-section and they will be moving parallel to the sides. Under these conditions, the resistance of a wire of a given material is directly proportional to the length L and inversely proportional to the cross-sectional area A. Thus we may write

$$R = \rho \frac{L}{A} \tag{120-1}$$

or

$$\rho = R/(L/A) \tag{120-2}$$

where ρ is a factor of proportionality constant with respect to L and A. The value of ρ for any material may be computed by Eq. (120-2) from the measured resistance of a given piece of wire of that material.

The quantity ρ as determined by Eq. (120-2) for a particular material is known as the resistivity of that material. Since ρ is constant with respect to L and A, it follows that once the value of ρ for a given material is found from measurements made with one particular piece of bar or wire, the resistance of any other piece of the same material can be computed by using this value of ρ in Eq. (120-1).

121. Metric Units of Resistivity. It follows from the definition of resistivity as given in Eq. (120-2) that ρ may be expressed in terms of some unit of resistance multiplied by some unit of area and divided by some unit of distance. If R is expressed in ohms, A in cm^2 and L in cm, ρ will be expressed in ohm-cm. For example, if a wire 800 cm long with a cross-sectional area of .01 cm^2 is found to have a resistance of 4 ohms, the resistivity of that material must be given by

$$\rho = \frac{4 \text{ ohms x .01 cm}^2}{800 \text{ cm}} = 5 \times 10^{-5} \text{ ohm-cm.}$$

122. English Units of Resistivity. In the English system, the unit of length commonly used in connection with resistivity is the foot, while the unit of area is a special unit equal to the area of a circle one mil (.001 inch) in diameter. This unit of area is called the circular mil and is abbreviated C.M. Thus if a wire 2 ft long has an area of 9 C.M. and a measured resistance of 20 ohms, the resistivity of that material will be given by

$$\rho = \frac{20 \text{ ohms x 9 C.M.}}{2 \text{ ft}} = 90 \text{ ohms-C.M. per ft.}$$

By using the C.M. as a unit, the computation of the area of a circular wire from its measured diameter is very simple because the area in C.M. is numerically equal to the square of the

diameter in mils.[1] For example, let us compute the resistance of a wire 15 ft long with a diameter of .020 inches, made from a metal having a resistivity of 12 ohm-C.M./ft. Since the diameter is 20 mils, we know immediately that the area is $(20)^2$ C.M. Substituting this in Eq. (120-1) gives

$$R = 12\ \frac{\text{ohm-}\cancel{\text{C.M.}}}{\cancel{\text{ft}}}\ \frac{16\ \cancel{\text{ft}}}{400\ \cancel{\text{C.M.}}} = .48 \text{ ohms.}$$

A similar calculation will show that a wire of the same material 1 ft long and one mil in diameter will have a resistance of 12 ohm. Thus in general, the resistance in ohms for any wire 1 ft long and 1 mil in diameter will be numerically equal to the resistivity of the material.

The C.M. can be used as a unit of area in dealing with wires having a rectangular cross-section. Any area expressed in square mils can be converted to C.M. by using the relation $\pi/4$ sq. mils. = 1 C.M.

123. Resistivities of Various Materials. The resistivities of various materials are given in the first column of the following table.

Table 123

Resistivity and Temperature Coefficients of Resistance

Material	Resistivity in ohm-cm at 20°C.	Temp. Coef. of Resistance	Material	Resistivity in ohm-cm at 20°C.	Temp. Coef. of Resistance
Aluminum	2.8×10^{-6}	.0039	Mercury	95.8×10^{-6}	.00089
Brass	$7. \times 10^{-6}$	.002	Nichrome	$112. \times 10^{-6}$	.00016
Carbon	$3470. \times 10^{-6}$	-.0005	(Ni 60, Fe 25, Cr 15)		
Constantin	$45. \times 10^{-6}$	$<.00001$	Nickel	7.8×10^{-6}	.006
(Cu 60, Ni 40)			Platinum	$10. \times 10^{-6}$	.003
Copper	1.7×10^{-6}	.0038	Silver	1.6×10^{-6}	.0038
Gold	2.4×10^{-6}	.0034	Sodium	5.0×10^{-6}	
Iron	$10. \times 10^{-6}$	.005	Tungsten	5.5×10^{-6}	.0045
Manganin	$44. \times 10^{-6}$	$<.00001$			
(Cu 84, Mn 12, Ni 4)					

The numerical value of the resistivity in ohm-C.M./ft=6.02×10^6 times the numerical value of the resistivity in ohm-cm.

It may be noted that pure metals have relatively low resistivities compared to the alloys. Silver, gold and copper are the three best conductors of all known materials. Nichrome is an alloy of nickel, iron and chromium, which is widely used in heating elements of such things as toasters and soldering irons. It is advantageous because its high resistivity permits a given resistance to be had with a relatively short and rugged piece of wire, and because it oxidizes slowly even at a red hot temperature. Rods of non-metallic conductors such as silicon carbide are often used in electric stoves and furnaces. A heating element made of a high resistivity material like this will be relatively wide and short, and hence more rugged.

1. This follows because the areas of any two circles are proportional to the numerical ratio of the square of their diameters. Thus

$$\frac{A}{1 \text{ C.M.}} = \frac{\text{Area of circle of diameter d mils}}{\text{Area of circle of diameter 1 mil}} = \frac{d^2}{1^2}$$

and

$$A = \frac{d^2}{1} \text{ C.M.}$$

The resistivities of insulating materials such as glass can be defined in the same way as the resistivity of a conductor. Thus, one kind of glass has a resistivity of 9×10^{13} ohm-cm. Such data however must be used with care in practice, because the conduction along the surface of a given insulator due to moisture and other impurities may be many times greater than the computed conduction through the material itself.

124. The Distribution of Charges in a Conducting Wire. It was stated above that the current will be uniformly distributed over the cross section of a long wire when a steady current is moving parallel to the sides. This may appear as a contradiction of the fact that excess charges on a conductor reside on the surface. It is true that the excess charges responsible for the difference in potential along a conductor will be on the surface. The charges that participate in the current however, are not excess charges. Rather they are the so-called "free" electrons which are uniformly distributed among the fixed atoms with which they are associated. The atoms are left with a positive charge for each electron that becomes free. The free electrons therefore constitute a cloud of movable negative charges diffused through a fixed distribution of positive charges, without giving an excess of either kind of charge inside the material. If we assume there is one free conduction electron in a conductor for each atom, the number of electrons that may be involved in conduction through a wire will be millions of times larger than the number of excess charges required on the surface to maintain the potential difference.

For alternating currents of high frequency, there is a so-called skin effect that limits the flow of current to the outer part of the cross-section. The resistance of a solid wire will accordingly be higher for such currents than it is for steady currents.

125. The Effect of Temperature on Resistance. The resistance of a conducting path usually depends on the temperature of the path. For some types of conductors such as carbon, the resistance decreases as the temperature increases. For other conductors, including metallic conductors the resistance increases with a rise in temperature. In general, the behavior of any given type of path with respect to temperature must be determined by experimental measurement. For most metals the rate of increase of the resistance with respect to the temperature is nearly enough uniform that we can use a constant temperature coefficient of resistance to describe this behavior approximately. This temperature coefficient of resistance is defined as the increase in resistance per unit of resistance at 0°C per degree rise in temperature. Thus if R_0 is the resistance at 0°C,

$$\alpha = \frac{\Delta R}{R_0\,\Delta t} \tag{125-1}$$

If R_1 is the resistance at any centigrade temperature t_1, then $\Delta t = t_1 - 0$, and $\Delta R = R_1 - R_0$ so that we may write

$$R_1 = R_0\,(1 + \alpha t_1) \tag{125-2}$$

For any other temperature t_2 we can write $R_2 = R_0\,(1 + \alpha t_2)$ and this equation can be combined with Eq. (125-2) to give

$$\frac{R_1}{R_2} = \frac{(1 + \alpha t_1)}{(1 + \alpha t_2)} \tag{125-3}$$

For the pure metals which are good conductors of electricity, such as silver, copper, and gold, the increase in resistance per degree rise in temperature is approximately 1/273 of its value at 0°C for all these metals. This rate of change is approximately the same as the rate of change of the pressure of an ideal gas with an increase in temperature. The values of α for these and various other metals are given in the second column of Table 123 on page 37. It will be noted in general that alloys such as constantin and manganin may have very small temperature coefficients as compared to those of pure metals. Because of the low temperature coefficient of alloys like constantin and manganin, these materials are commonly used in making standard resistance coils. Such coils will have very nearly the same resistance at all temperatures.

As defined above, α will not be found exactly constant for all values of t. When very precise measurements must be made or when a wide range of temperatures is involved, the behavior of a resistance can not be adequately described by the use of a single constant. In such situations Eq. (125-1) is not used at all, and the equation

$$R_t = R_0 (1 + at + bt^2 + ct^3) \tag{125-4}$$

is used instead of Eq. (125-2). In this equation, a, b, and c are constants which must be determined by experiment for any given material. The terms containing b and c amount to small correction terms which become relatively more important at higher temperatures.

126. Resistance Thermometers. A coil of wire having a high temperature coefficient of resistance may be used as a resistance thermometer, since the measured change in resistance of such a coil will indicate any change in temperature of the coil. Nickel and platinum are commonly used for this purpose because they have relatively large temperature coefficients and do not corrode readily. Note that a resistance thermometer made of an alloy like constantin would be very insensitive.

Chapter 13

CONDUCTION THROUGH LIQUIDS

130. The conduction of electricity through some liquids is due chiefly to the rather free motion of temporarily unattached electrons, and therefore does not differ essentially from conduction in solids. This is particularly true of molten metals. The metal mercury, being liquid at room temperature, is a typical conductor of this type. There is however another type of liquid conductor in which the conduction is due to the motion of charge atoms through the body of the liquid. Such conduction is of particular interest because of the associated chemical effects, and it will be discussed in this chapter.

131. Electrolytic Dissociation. Many chemical compounds, when dissolved in water suffer a dissociation of their molecules into two parts which are equally and oppositely charged. For example, when ordinary salt dissolves, its molecules dissociate into their separate atoms except that each chlorine atom takes with it one of the electrons which belongs to the corresponding sodium atom. Thus a solution of common salt contains negatively charged chlorine atoms and positively charged sodium atoms suspended in the solvent. Many other chemical compounds dissociate in this way, including particularly the various salts of the metallic elements. Solutions containing charged particles resulting from dissociation are called electrolytes.

132. Ions. The charged particles which result from electrolytic dissociation are called ions. The name comes from the Greek verb which means to go, and it signifies the tendency of the ion to move when placed in an electric field. In general, any molecule or atom that has become charged by losing or gaining an electron, is referred to as an ion. Thus in addition to the electrolytic ions being considered here, there are other types of ions which occur under other circumstances.

Electrolytic ions include single atoms of various elements which have gained or lost an electron, such as the sodium and chlorine ions mentioned in the preceding section. Other electrolytic ions consist of several atoms which stick together and behave as a single particle. Thus when a molecule of silver nitrate ($AgNO_3$) is dissociated, there will be one silver ion (Ag^+) consisting of a silver atom which has lost one of its electrons and a nitrate ion (NO_3^-) consisting of one nitrogen atom and three oxygen atoms sticking together. This ion behaves as a single particle having an excess charge of one electron. Copper sulphate ($CuSO_4$) breaks up into a copper

ion Cu^{++}, which is a copper atom that has lost two electrons, and a sulphate ion (SO_4^{--}) which consists of one sulphur and four oxygen atoms with two extra electrons. A group of atoms behaving as a single particle in chemical reactions is called a radical.

133. Valence. The number of electronic charges which have been lost or gained to give an electrolytic ion its charge is referred to as the valence of the ion. Valences may be positive or negative, depending on the sign of the resultant charge left on the ion. Thus sodium is said to have a valence of +1, because an atom of sodium loses one electron when it goes into solution. The SO_4 radical normally gains two electrons when it becomes an ion in a solvent and it is said to have a valence of -2.

It follows from above that the charge q on any ion will be some multiple of the electronic charge e, and thus we may write

$$q = ve, \tag{133}$$

where v is a whole number representing the valence.

The valence of a chemical element as exhibited in a solution is the same as the chemical valence which determines the way in which the chemical elements combine to form compounds. This is to be expected since the dissociation which occurs in a solution is just the reverse of the process of chemical combination. In general chemical combinations occur between substances with positive valence and substances with negative valences so that the resultant molecules are electrically neutral. Hence positive and negative charges appear in equal numbers when molecules are dissociated.

It may be noted here that metal atoms as a class are all electropositive in that they tend to lose one or more of their outer electrons to form positive ions. This same behavior is characteristic of metals in the solid state, where their exceptional conductivity is attributed to the fact that some of the outer electrons of the atom are relatively free to move from one atom to the next.

134. Atomic Weight. Atoms of different substances have different weights. All atoms of a kind weigh the same with the exception of isotopes, which will not be considered here. Since atoms are so small and so numerous, they are often counted and weighed in groups of N atoms each, where $N = 6.023 \times 10^{23}$. In other words, we find it convenient to deal with atoms by figuratively grouping them in standard packages with 6.023×10^{23} atoms of a kind in each package. One result of this is that if an oxygen atom is sixteen times as heavy as a hydrogen atom, a standard package of oxygen atoms will be sixteen times as heavy as a standard package of hydrogen atoms. The number N is called Avogadro's number.

The atomic weight A of any substance is the numerical value of the weight of N atoms of that substance in grams. Because of the unfortunate but established use of the term "atomic weight" the student may easily mistake the atomic weight of a substance to mean the weight of an individual atom. Instead it requires N atoms to weigh A grams, and hence the mass m of one atom will be given by the equation

$$m = \frac{A \text{ grams}}{N} \tag{134}$$

The necessity of writing grams after A to obtain mass arises from the definition of A as a pure number.

The value of Avogadro's number was arbitrarily fixed in an indirect manner when a weight of 16 grams was chosen as the weight of a standard mass of oxygen for use in chemistry. N was thus fixed as the number of oxygen atoms in that amount of oxygen.

When an atom is converted into an ion by losing or gaining a few electrons, the weight of the ion will be the same as that of the atom to within three significant figures. This is so because the mass of an electron is so small compared to the mass of any atom.

135. Electrolysis. If two conducting strips, called electrodes, are placed in an electrolyte at some distance apart, these electrodes can serve as the terminals of a conducting path through

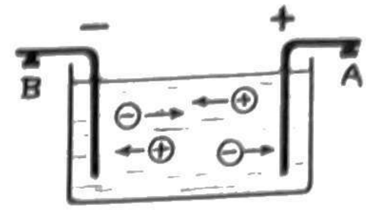

Fig. 135

the body of the electrolyte. When a difference in potential is applied between the two electrodes, the associated electric field will cause the positive ions in the electrolyte to move "down" through the potential gradient to the negative electrode. At the same time, this field will also cause the negative ions to move "up" through the potential gradient to the positive electrode. The current through the conducting path will then be the resultant of both of these motions. A vessel of electrolyte with two electrodes as described above is referred to as an electrolytic cell. The general term electrolysis is applied to any chemical process which occurs in an electrolytic cell due to the transporation of charge through the cell.

136. Electrolytic Deposits. When an ion reaches an electrode of the opposite sign, the charge on the ion will in many cases pass over to the electrode leaving the ion to exist as an ordinary atom. If the atom is a metal atom, it may thereby be deposited on the electrode along with other similar atoms to form an adherent layer of metal. Surface layers of copper, silver, chromium, etc., may thus be deposited, or plated, on any conductor which is used as an electrode.

A metal which is deposited by electrolysis may be in the form of a compact, adherent, layer having physical properties just like any ordinary piece of that metal. This type of deposit is of course particularly desired in commercial plating. Depending on the kind of metal and the conditions under which it is deposited however, the layer of metal may be rather porous or spongy. In some cases the deposit may even build up in the form of long filaments extending out from the electrodes with branches resembling the branches of trees.

When ions of gas-forming elements lose their charges at an electrode, then electrolysis may result in the production of bubbles of that gas at the surface of the electrode. Thus hydrogen gas will be liberated at the negative electrode of a cell whenever electrolysis occurs in an acid solution containing hydrogen ions. The term hydrogen ion is used here in the interest of simplicity to refer to what is probably a positively charged particle consisting of one molecule of water attached to a hydrogen ion.

137. Number and Mass of Ions Deposited. If a current I flows through an electrolytic cell for a time t, the total charge Q that has moved through the cell will be given by the equation

$$Q = It \tag{137-1}$$

The total number of positive ions that must have arrived at the negative electrode will accordingly be equal to the total charge Q divided by the charge q on each ion. Since $q = ve$, we may write

$$\text{number of ions deposited} = \frac{Q}{ve} \tag{137-2}$$

The number of negative ions that must have gone up to the positive electrode to have their charges neutralized can be computed in the same way using the magnitude of Q for the magnitude of the total charge on all the negative ions involved. This holds because if a total positive charge Q passes from the positive electrode into the electrolyte in the direction of the conventional current, it would neutralize an equal total negative charge brought up to the electrode from the other direction by the negative ions.

For either positive or negative ions, the total mass deposited will be equal to the number of ions times the mass m of each ion, so that we may write

$$M = (Q/ve)\,m \tag{137-3}$$

Substituting for m from Eq. (134) above, we may therefore write

$$M = \frac{(A \text{ grams})}{N\,ve}\,Q \tag{137-4}$$

By using $Q = It$, this may also be written

$$M = \frac{(A \text{ grams})}{v\,Ne}\, It \tag{137-5}$$

The product Ne which appears in Eq. (137-5) is the amount of charge carried by N electrons. It is sometimes used as a unit of charge called the Faraday. To within three significant figures it is equal to 96,400 coulombs.

138. Electrochemical Equivalent. In order to deposit a given mass M of a certain chemical element by electrolysis, a certain charge Q must be transported up to the electrode because each ion has a fixed amount of charge q associated with a fixed amount of mass m. If x ions are deposited, the total mass M will be xm, and the corresponding total charge Q will be xq. Thus the quotient M/Q will be the same as m/q for any value of x since

$$\frac{M}{Q} = \frac{xm}{xq} = \frac{m}{q} \tag{138-1}$$

In other words, the ratio of M to Q for a given element will always be the same regardless of whether a large or a small mass is deposited.

The constant ratio of M/Q for any given element is called the electrochemical equivalent for that element. Since $Q = It$, the definition of z could be written

$$\boxed{z = \frac{M}{It}} \quad \text{or} \quad M = zIt \tag{138-2}$$

A comparison of Eq. (138-2) with Eq. (137-5) shows that

$$\boxed{z = \frac{(A \text{ grams})}{v\,Ne}} \tag{138-3}$$

The relationship expressed in Eq. (138-3) can also be derived by using the equations $M/Q = m/q$, $m = A \text{ grams}/N$ and $q = ve$.

139. The Measurement of Current by Electrolysis. The average value of a current flowing through an electrolytic cell for a given time can be determined by applying Eq. (138-2) in the form

$$I = M/zt$$

The value of z is known for all metals, the mass can be determined by weighing and the time can be easily observed. This method, while inconvenient, is comparatively accurate, and it can be used to calibrate the more convenient moving-coil ammeters. Silver deposited from a solution of silver nitrate is often used for such measurements.

Chapter 14

CONDUCTION THROUGH LIQUIDS (continued)

140. The term electrolysis (Sect. 135) applies in general to any chemical action which occurs when a current passes through an electrolytic cell. If both electrodes are made of the same material, an external source of power such as a mechanical generator will be required to cause a current to flow through the cell. Thus the external source expends energy to make the chemical action take place. In other cells where the electrodes are made of different material, the electrolytic cell constitutes a voltaic cell. Such a cell is itself capable of driving a current through an external conducting path connecting the two electrodes together, and here we may consider that the chemical action furnishes the energy to keep the current flowing. Whether the current produces the chemical action or vice versa, there is always a definite relationship between the current and the mass of material involved in the reaction as expressed by Eq. (137-5). The simple deposition of atoms of metals in electroplating as described in Sect. 136 in one example of electrolysis. Other examples will now be considered.

141. Electrolysis of Copper in a $CuSO_4$ Cell. In many electrolytic processes, the neutralization of the charge on an ion at an electrode does not result in a simple deposit of the substance involved. Instead, the neutralized ion may react chemically with either the material of the electrode or the solvent. The electrolysis which occurs in a solution of copper sulphate ($CuSO_4$) with copper electrodes furnishes an example in which the neutralized negative ion reacts with the positive electrode.

The molecule of copper sulphate dissociates into a copper ion with a valence of +2 and a sulphate ion (SO_4) with a valence of -2. When the sulphate ion is neutralized at the positive electrode, a chemical reaction occurs which takes an atom of copper and the equivalent of two positive electronic charges from the electrode and returns the sulphate ion to the electrolyte just as it was before. The net result in such a process is that the total number of sulphate ions in the solution remains constant while copper ions are dissolved from the positive electrode. These ions carry positive charge away from the positive electrode, and they move toward the negative electrode where they will be deposited.

Since the dissolution of the positive electrode in such a process must serve to replenish the metal ions in the solution, it is necessary to use a positive electrode made of the same metal if a considerable amount of metal is to be deposited on the other electrode. Generally the metal deposited on the negative electrode will be quite pure, whether the metal dissolved from the other electrode is pure or not. Thus electrolytic process may be used to refine metals.

142. The Electrolysis of Water. The electrolysis which occurs when a current is sent through a dilute solution of sulphuric acid (H_2SO_4) in water with a relatively inactive electrode furnishes another example where the liberated negative ion reacts with the solvent. To begin with, each dissolved molecule of sulphuric acid gives two positive hydrogen ions with a valence of one, and one sulphate ion (SO_4) with a negative valence of two. When a current is sent through the cell, the positive hydrogen ions go to the negative electrode where they are deposited as bubbles of hydrogen without any complications. The negative sulphate ions go to the positive electrode where they lose their negative charge. As soon as a sulphate ion is neutralized, it breaks up a molecule of water, takes two electrons from it, and liberates the oxygen atom. This leaves two hydrogen ions and the original sulphate ion in the solution, while oxygen gas is liberated in bubbles at the surface of the positive electrode. The net result is just as if the water molecules had dissociated by themselves into hydrogen and oxygen ions, and as if these ions had gone to the respective electrodes where the gases were liberated without the dissolved acid playing any part in the process. It should be noted, however, that this action will not take place in pure water. However, the simplified equivalent concept of the electrolysis of water in a solution of sulphuric acid can be used in computing the mass of hydrogen or oxygen that will be liberated by a given

current in a given time. In other words the computation can be made on the assumption that each water molecule gives two hydrogen ions, each with a valence of +1, and one oxygen ion with a valence of -2, and that these ions go to their respective electrodes for liberation.

To compute the volume of hydrogen or oxygen liberated in electrolysis after the mass is known, it is convenient to remember that both gases are diatomic, and that N molecules of any gas will occupy 22.4 liters of volume at $0^\circ C$ under a pressure of 1 atmosphere.

143. Faraday's Laws of Electrolysis. The dependence of electrolytic chemical action on the flow of charge through the cell was recognized experimentally by chemists before much was known about the structure of individual atoms and ions. Two laws formulated by Michael Faraday (1797-1867) describe the relationship between the chemical and electrical quantities involved. Faraday's first law states that the mass of any given substance deposited by electrolysis is proportional to the charge passing through the cell. The second law states that when the same current passes through several different electrolytic cells, the masses deposited in the various cells will be proportional to the respective chemical combining equivalents. These laws are convenient for chemists who are already familiar with the concept of chemical combining equivalent. For others they convey no information that has not already been expressed in mathematical form in Eq. (137-5) of Sect. 137.

144. Electrolytic Action in Voltaic Cells. A voltaic cell as described in Chap. 7 consists essentially of two conducting electrodes of different materials immersed in an electrolyte. Inside the cell, an equivalent chemical force acts to remove positive charges from the negative electrode and place them on the positive electrode, thereby creating a difference in potential. The chemical action stops when the chemical force is balanced by the potential difference it has built up. Now if the electrodes are connected by a conducting path outside the cell, a current will flow through this path from the high potential positive electrode to the negative electrode. This current will tend to reduce the potential difference between the electrodes. The equivalent chemical force will then no longer be balanced, and it will start carrying positive charges to the positive electrode inside the cell. To maintain a steady flow of charge from plus to minus potential in the path outside the cell, the chemical force must act as a pump and carry charges from minus to plus potential inside the cell at the same rate. Thus it is that the chemical action furnishes the energy to keep the current flowing. As in all electrolytic actions, the mass of material involved in the chemical action will bear a definite relationship to the current, and no chemical action will take place when no current is flowing.

One of the results of electrolytic chemical action which occurs when a voltaic cell furnishes a current is usually a dissolution of the negative electrode. The continued use of such a cell may completely dissolve all the material of that electrode. Such a dissolution represents a loss of chemical energy. This energy is transformed into electric energy and is used to drive the current.

145. Polarization and Local Action in Voltaic Cells. The motion of charge through a voltaic cell and the chemical actions which result when the cell furnishes a current may alter the constitution of the electrolyte in any given region. The same actions may also deposit foreign atoms which cover the surface of the electrodes, as is the case when bubbles of hydrogen are deposited on a metal electrode. In general, such effects tend to reduce the potential difference which the cell can furnish, and they are referred to as polarization effects. They are often of a temporary nature and may disappear of their own accord if the cell is left standing without any current flowing.

Another difficulty that is encountered in the use of voltaic cells is due to the almost inevitable presence of impurities in the electrodes. Such impurities often give a localized electrolytic action. A small piece of impurity in contact with an electrode and with the solution constitutes a small voltaic cell located on the surface of the electrode. Since the impurity is in contact with the electrode, the minature cell sets up a localized circulation of charge. Thus electrolytic action may eat away an electrode although the cell as a whole is not furnishing any useful current.

Such local actions reduce the useful life of commercial cells and they may become worthless in time without ever having been used.

146. Standard Voltaic Cells. Since the emf of a voltaic cell depends on the combination of materials used, a standard cell having a known emf can be made by using carefully purified materials. Such cells serve as convenient standards of potential which can be used to measure unknown potential differences by comparative methods.

The Weston normal standard cell is designed to give a fixed emf that will remain constant in use provided it delivers only very small currents for brief periods of time. One electrode of this cell is a paste of mercurous sulphate in contact with mercury held in the bottom of one side of an H-shaped glass vessel. The other electrode is an amalgam of cadmium held in the bottom of the other side of the glass vessel. The electrolyte connecting the two electrodes is a saturated solution of cadmium sulphate with crystals of cadmium sulphate to keep it saturated. This cell has an emf of 1.01830 volts at 20°C. The value of such a cell as a standard may be seen from the number of significant figures given for its emf.

The emf of a saturated cadmium cell depends upon the temperature. The emf of a cell in which the solution is not saturated is practically independent of the temperature, but it will be different for different cells. Unsaturated cells are accordingly convenient to use, but each one must be calibrated individually.

147. Primary and Secondary Cells. By using a stronger outside source of power, an electric current can be made to flow through a voltaic cell in a direction opposite to the direction of the current the cell itself would furnish. In other words, a stronger outside source of energy can make a current flow through the cell in opposition to the internal equivalent chemical force. In some cells, this reversed current causes a chemical action which is the reverse of the chemical action that occurs when the cell furnishes current. After such a cell has expended its chemical energy in use, it can accordingly be restored to its original condition by sending a reversed current through it. Such a cell is called a storage, or secondary cell.

A voltaic cell which cannot be restored after use by a reversed current is called a primary cell. In these cells, a reversed current produces a chemical action in the cell, but the action is not the reverse of the chemical action which occurs when the cell furnishes a current.

148. Lead Storage Cells. Commercial lead storage batteries like those used in motor cars offer a good example of the behavior of storage cells in general. These cells have a negative electrode of lead and a positive electrode of lead peroxide. The electrolyte is a dilute solution of sulphuric acid. When such a cell furnishes a current, the net result of the associated chemical action is to reduce the lead peroxide on the positive electrode and to deposit lead sulphate on both electrodes. At the same time, the concentration of sulphuric acid in the electrolyte is decreased, as is indicated by a decrease in the specific gravity of the electrolyte. In this final state, the chemical materials in the cell have less energy than they had before. Thus when a storage cell is furnishing a current, charges are moving through the cell in the direction of the chemical force, and chemical energy is being converted into electrical energy. When all the lead peroxide has been reduced, no more chemical energy is available and the cell is said to be discharged.

A lead cell is recharged by using a stronger external source to send a reversed current through it. This reversed current reverses the chemical action which occurred while the cell was being discharged, and restores the cell to its original condition. Note that while a cell is being recharged, charges are being forced through the cell against the equivalent chemical force. Thus the electrical energy furnished by the external source of power is being converted into chemical energy and stored in the battery in that form.

A lead storage cell as described above gives a potential difference of about 2.1, depending on the state of charge. Most automobile batteries have three such cells connected together in a unit to give a potential of about 6.4 volts.

A very simple lead storage cell can be formed by using two lead strips as electrodes in a cell containing dilute sulphuric acid. A brownish deposit of lead peroxide can be formed on one

of them by sending a current through the cell from an outside source. The cell then can be discharged and recharged like the commercial cells described above.

Chapter 15

SOURCES OF ELECTRIC POWER

150. Electromotive Force. An agent that can drive electric charges around a conducting circuit is referred to as an electromotive force, or emf. In most circuits the emf is concentrated in a short path that forms a part of the whole circuit. An emf acting along such a path will do work on any charge that moves along the path in the direction of the driving force. By definition, the magnitude of an emf is the amount of work it will do per unit charge on any charge that passes through the path along which the emf acts. Since an emf by definition must be able to do work on charges, an emf must be a source of energy. A path containing an emf is analogous to a pipe containing a pump to impel a fluid along the pipe. Thus an emf may be thought of as an electrical pump. A voltaic cell where chemical action impels positive charges through the cell is a good example of an emf.

The definition of electromotive force E may be stated in the form of the equation

$$E = W/Q \tag{150}$$

where W represents the work done on a charge Q when that charge passes through the path along which the force acts. Note that the term electromotive force is thus used in a technical sense to refer to an amount of work per unit charge, instead of referring directly to the force which does this work. It is more natural to use the term emf to refer to the force itself, and this literal usage of the term inevitably persists. Fortunately little confusion will arise from this dual usage in practice if we merely remember that an emf must be measured in terms of the work done per unit charge whenever we wish to assign a numerical value to it or use it in a mathematical equation.

Whenever the term emf is used in its literal sense to refer to the force that does the work, it is considered to act in the direction in which a positive charge is impelled to move along the path containing the emf.

151. Units of Electromotive Force. Since the electromotive force in a path is defined as an amount of work done per unit charge passing through the path, any unit of work divided by any unit of charge may be used as a unit of emf. For example, if the chemical emf of a storage battery does 60 joules of work on 10 coulombs of charge passing through it, the emf of the cell is

$$E = \frac{60 \text{ joules}}{10 \text{ coulomb}} = 6 \text{ volts}$$

152. Emf as a Source of Electrical Energy. An emf can serve as a source of electrical energy because it can exert a driving force on moving charges. In this respect, its action may be illustrated by the simple circuit previously considered in Sect. 92. The essential details of that circuit are reproduced in Fig. 152 for more convenient reference. The source of power C which drives the charges around in the circuit can now be referred to as an emf according to the definition given above. A two-stage transformation of energy occurs in such a circuit. For example, let us suppose the source of energy is a chemical cell. The first stage of the transformation occurs as the charges are carried up through the cell by the chemical force; this converts chemical energy into electric energy, which is

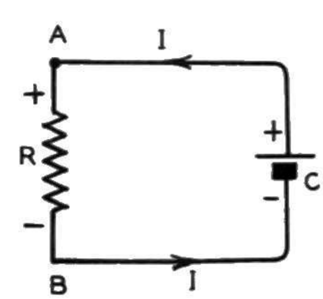

Fig. 152

carried out of the cell by the charges. The second stage occurs as the charges move down through the resistance R; there the electrostatic energy is converted into heat.

The energy per unit charge (V) given out in the resistance is equal to IR. This energy per unit charge must also be equal to the work done per unit charge by the chemical emf if there is no loss in the cell itself. In that special case we can therefore write

$$IR = V = E. \tag{152}$$

Note that when an emf is driving a current in this way, the charges are taken in at a low potential and delivered at a high potential. In other words they move through the emf from low to high potential. This is in contrast to the flow of charge through a resistor, where the charges always move from high to low potential.

153. Energy and Power Furnished by an Emf. It follows from the definition of emf that if a charge Q flows in a time t through a path containing an emf E, the work W done by the emf will be given by

$$W = EQ \tag{153-1}$$

If the current is constant, $Q = It$, and hence

$$W = EIt. \tag{153-2}$$

The power P furnished by the emf will be W/t, so that

$$P = EI. \tag{153-3}$$

154. Kinds of Emf and Comparison of Emf with Potential Difference. There are different kinds of emf which may be classified according to the source of energy. For example the chemical emf in a voltaic cell furnishes electrical energy from chemical energy. The electro-mechanical emf of a rotating generator furnishes electrical energy from mechanical energy. Other types of emf to be studied later will include photo-voltaic and thermal emfs. These emfs transform light energy and thermal energy respectively into electric energy.

Potential difference and emf are so much alike that the distinction between them is not immediately obvious. They are similar in that both involve a force that can act on a movable charge, and both can furnish energy to make charges move. With regard to the force, there is a distinction in that the electrostatic forces associated with a potential difference can never make charges circulate around in a circuit; they can only drive them from one side of the circuit to the other. With regard to the energy, there is a distinction in that the energy furnished by a potential difference already belongs to the charge as electrostatic potential energy, while the energy furnished by an emf comes from an external source. The energy from a potential difference is analagous to money a person withdraws from his own bank account in that it already belongs to him, while the energy from an emf is analagous to income from an external source. Physically the money is the same, but the amounts belong in different categories. Thus, in spite of the distinction between potential difference and emf, they can both be measured in volts, and they can be added, subtracted, or equated in mathematical expressions.

155. Internal Resistance in an Emf. A usable emf consists of a conducting path along which a force acts on electric charges. In any actual emf, this conducting path will have a certain amount of resistance, which is referred to as the internal resistance of the emf.

The internal resistance of voltaic cells is mostly in the electrolyte, and it depends in general on the size and spacing of the electrodes. For example, a very small flashlight cell may have an internal resistance of about one ohm. On the other hand, a larger dry cell made of the same materials and having the same emf may have an internal resistance of only .05 ohms. The lead cells in the storage battery of an automobile may have an internal resistance as low as .005 ohms. Rotary mechanical generators may have large or small internal resistances, depending on the size and length of the wire used to make the coil in which the emf is generated.

156. Terminal Voltage of an EMF Producing Electric Energy. As stated above, an emf furnishes energy to charges that move through it in the direction of the acting force. If no energy is lost along the path, the charges will emerge from the emf with an increase in electrostatic energy per unit charge that is exactly equal to the work done per unit charge by the emf. In other words, the gain in potential will be equal to the emf. If there is internal resistance along the path where the emf acts, some energy will be lost in heat. The net gain in potential will then be something less than the emf. Since the loss of potential due to an internal resistance R_i will be IR_i, we may write

$$\text{(Net Gain in Potential)} = \text{(Gain from emf)} - \text{(Internal } IR_i \text{ loss)}. \tag{156-1}$$

The net gain in potential as given by the above equation is often referred to as the terminal voltage of the emf.

In adding the changes in energy which are encountered by a charge passing through an emf, the changes may be added in any order just so all of them are taken into account. For example, although the resistance in a voltaic cell may be distributed all along the path from one terminal to the other, we may consider this resistance to be concentrated at one point. Also regardless of where the emf actually acts along the path, it can be considered to be concentrated at some one point, as illustrated in Fig. 156-1. Assuming this simplified equivalent distribution of the emf and internal resistance, a graph of potential is shown in Fig. (156-2) to represent the gains and losses encountered in passing through a cell for the case where Eq. 156-1 applies. The abrupt rise labeled E represents the gain from the emf, and the more gradual drop labeled IR_i is the loss due to the internal resistance; V represents the net gain. An emf furnished by a generator is usually represented in circuit diagrams by the symbol − +

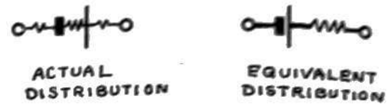

Fig. 156-1

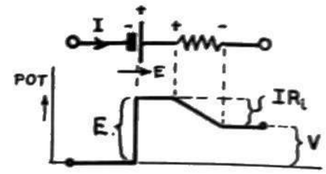

Fig. 156-2

Positive and negative signs used in diagrams on the terminals of cells and generators indicate the relative potentials which would exist if no current was flowing. This means that the positive terminal is the one toward which the emf is directed; it may or may not be at the higher potential in any given application.

Using conventional symbols for the quantities involved in Eq. (156-1), we may write

$$V = E - IR_i. \tag{156-2}$$

For many sources of emf, E and R_i will be approximately constant for different values of I. Equation (156-2) therefore gives the values of a dependent variable V in terms of an independent variable I and two constants E and R_i. This equation may be regarded as a complete specification of the electric behavior of a path containing an emf, just as the equation V = IR describes the electric behavior of a simple resistive path. It is to be noted that the terminal voltage will be equal to the emf only when no current is flowing.

157. Terminal Voltage of an Emf Receiving Electric Energy. As pointed out above, an emf must be able to serve as a source of electric energy if it is to be able to exert a driving force on moving charges. It also follows that an emf will receive electric energy if charges are forced through it in a direction opposite to the emf by some stronger source. Sources of emf are able to receive electric energy in this way by virtue of the fact that they can convert it into some other form. For example, the equivalent chemical force in a storage cell can receive energy by converting it into chemical energy. The emf of a mechanically driven generator can receive energy by operating as an electric motor converting the electric energy into mechanical energy. As will be explained in Chap. 31, a given machine can operate either as a generator or motor, depending on whether the current moves through the machine with the generated emf or against it.

When an emf receives energy from charges moving through it, the loss of electrostatic energy per unit charge to the emf is equal to the emf. If the emf has an internal resistance R_i, the charges will also lose energy in the form of heat in the resistance. Regardless of the direction of a current through a resistance, the resistive heat loss is always the same. Hence we may write

(Net loss in potential) = (Loss to emf) + (Internal IR_i loss) (157)

It follows that the terminal voltage of an emf receiving electric energy must always be greater than the emf.

A graph of potential showing the losses encountered by a charge moving through a cell with the current against the emf is shown in Fig. 157, where the cell is represented in the equivalent simplified manner described in Sect. 156.

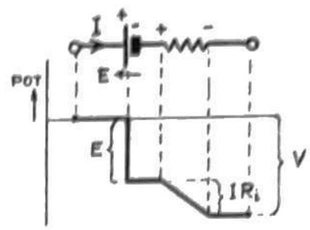

Fig. 157

Chapter 16

THE SERIES CIRCUIT

160. A number of electric paths connected in series to form an endless circuit constitutes a series circuit. For example, a voltaic cell connected to send a current through a lamp bulb is such a circuit. If there is no internal resistance in the cell, and if the connecting wires have no resistance, the drop in potential across the lamp will be equal to the emf as explained in Sect. 152. These ideal conditions never exist in practice although they may be closely approximated in some cases. In this chapter we shall consider the behavior of series circuits in general.

161. Series Circuit with one Emf. If a single emf E with an internal resistance R_i is connected in a series circuit with one or more resistances such as R_1 and R_2, a current I will flow around in the circuit in the direction of the emf. Any charge moving with the current will gain an amount of potential equal to the emf E of the cell when it passes through the cell. It will also lose potential energy in the form of heat when it passes through any of the resistances in the circuit, and the potential lost in any one resistance will be equal to the IR drop in that particular resistance. The sum of all the potential drops encountered as a charge goes once around the circuit must be equal to the energy per unit charge received by the charge in passing through the emf. In other words,

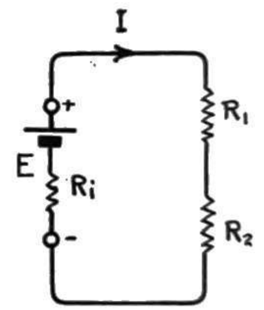

Fig. 161-1

$$IR_i + IR_1 + IR_2 = E \tag{161-1}$$

and hence

$$I = \frac{E}{R_i + R_1 + R_2} \tag{161-2}$$

Thus the current is equal to the emf divided by total series resistance of the circuit. After the current I has been computed, Eq. (161-1) can be used to find how much of the available potential is expended in each resistance.

In laboratory circuits, the resistance of the connecting wires is often negligible. In power circuits however, the connecting wires which form a part of the circuit may extend over miles of distance, and they may have appreciable resistance. For example, consider a generator with an internal resistance R_i attached to a stove of resistance R_2 through long lines having a total

resistance R_l, counting both sides of the circuit. In that case, IR_i would represent the part of the available potential which is lost in the generator itself, IR_l the potential lost in heat along the power lines and IR_s the potential V delivered to the stove. Note that V would then be less than E by an amount $IR_i + IR_l$.

162. Series Circuit with Several Emfs. If several emfs are connected in a closed series circuit with several resistances, the current will flow in the direction that gives the charges a net gain in energy from all the emfs involved. Thus if there are two emfs tending to drive the current around the circuit in opposite directions, the current will flow in the direction of the stronger emf. In passing through any opposing emf, a charge moving with the current will lose potential energy to that emf. Hence the net gain in energy per unit charge from all the emfs in a circuit will be the sum of all the emfs acting in the direction of the current, less the sum of all the emfs acting in the opposite direction.

The amount of current that will flow in a series circuit with several emfs will be given by the general relationship which may be written in the form

$$\text{Net gain in energy per unit charge from all the emfs} = \text{Sum of all the IR drops} \qquad (162\text{-}1)$$

If we let ΣE represent the net gain in potential from all the emfs we may write

$$\Sigma E = IR_1 + IR_2 + IR_3 + \cdots \qquad (162\text{-}2)$$

The internal resistances of the emfs must be included among the R's on the right hand side. By factoring out the I and letting R stand for the total series resistance in the circuit, we may write

$$I = \frac{\Sigma E}{R} \qquad (162\text{-}3)$$

One common example of a series circuit containing two opposing emfs is the circuit used to charge a storage battery from a mechanical generator. The battery and generator are connected so that the stronger emf of the generator drives the current through the battery against its chemical emf. Commercial storage batteries and generators ordinarily have rather low internal resistances and an additional resistance R_c is usually placed in the circuit to control the amount of current.

It follows from above that a number of cells may be connected together in series to get a larger emf. The total emf of such a battery of cells will be the sum of the separate emfs if all the emfs act in the same direction.

163. Ohmmeters. One type of commercial ohmmeter used to measure unknown resistances furnishes a practical example of a simple series circuit with a single emf. Such ohmmeters have a current meter, a cell, and an adjustable resistance R_1 connected in series between two external terminals. An unknown resistance to be measured is connected between these terminals to complete the circuit. The amount of current that flows will then indicate the magnitude of the unknown resistance. In this type of ohmmeter, a larger unknown resistance gives a smaller reading on the meter. The scales of such meters are ordinarily marked to indicate the value of the unknown resistance directly in ohms.

164. Measurement of an Emf by a Voltmeter. A voltmeter attached to the terminals of a source of emf indicates the terminal voltage of the emf. If a moving-coil voltmeter is used, it will take some current from the source of emf, and hence the observed voltage will be less than the emf we wish to measure according to Eq. (156-2). As stated in that equation, the difference between the observed terminal voltage V and the desired emf E depends on the current I taken from the emf. The current I in turn depends both on the internal resistance R_i of the emf and the resistance R_v of the voltmeter. The relationships involved can be written by noting that the voltmeter connected to the emf E forms a series circuit so that

$$E = I(R_i + R_v) \qquad (164\text{-}1)$$

The terminal voltage of the emf is also the voltage between the voltmeter terminals, so that

$$V = IR_v \tag{164-2}$$

It then follows that

$$\frac{V}{E} = \frac{R_v}{(R_v + R_i)} \tag{164-3}$$

This last equation shows that if the voltmeter resistance is much larger than the internal resistance of the emf, the observed terminal voltage will be approximately equal to the emf E.

The terminal voltage of a source as observed by an electrostatic voltmeter (Sect. 46) will give a true measure of the emf because such meters take no current except for a temporary current required to deflect the meter. These voltmeters are satisfactory for measuring high potentials, but in the portable form they are not sensitive enough to measure voltages less than a hundred volts. They also have a disadvantage in that the scale divisions are not uniformly spaced over the whole scale.

Chapter 17

ELECTRIC NETWORKS

170. In a network of paths containing emfs and resistances, the current in any one path will depend upon all the E's and all the R's in the whole network. If the E's and R's are known, it is possible to compute the unknown currents in all the paths. The general procedure is to use relationships already known to write a group of equations between the known E's and R's and the unknown I's, and solve these equations simultaneously. In general, unknown emf's and resistances can also be found, provided some of the currents are known so that the total number of unknown quantities does not exceed the total number of paths in the network.

The relationships used to give the required simultaneous equations include

1. Ohm's Law: $V = IR$ for a resistive path. (Sect. 93)
2. Kirchhoff's Law I: The algebraic sum of all currents flowing into a junction point is zero. (Sect. 105)
3. Kirchhoff's Law II: The algebraic sum of all changes in potential encountered in tracing through the several paths of any closed loop in the network must be zero when we return to the point from which we started.

Kirchhoff's second law is essentially the same as the law for potential differences in connected paths as given in Sect. 104. That law has merely been reworded here to apply to a sequence of steps that carry us back to the point from which we started. The changes in potential involved in this law include both changes encountered in passing through emfs and changes which arise from IR drops along resistances.

Included in Kirchhoff's Law II is the fact that the gain in potential from any one point in a network to any other point is the same for all paths that pass between the same two points.

In writing the simultaneous equations for a given network, unknown quantities are to be represented by letter symbols. If a current in a particular path is unknown, it will be necessary to assume a direction. Either of the two possible directions may be arbitrarily chosen. If your choice happens to be right, a positive numerical value will be found for that current when the equations are solved. If your choice of direction is wrong, a negative value having the same magnitude will be found for the current. In either case, you will have obtained the desired information about the magnitude and actual direction of the current.

171. Parallel Combinations of Voltaic Cells. Several cells may be connected in parallel as shown in Fig. 171 to furnish a large current I through a path R without having all the current come from one cell. An application of the network laws as given in the preceding section will show that all cells in such a parallel combination should have the same emf. If the emfs are not the same, wasteful currents may be sent backwards through the weaker cells, even when no current is being drawn from the combination as a whole.

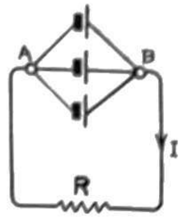

Fig. 171

172. The Potentiometer. A potentiometer is a network designed to give a variable known potential difference which can be used to balance an unknown voltage, and thereby measure the unknown voltage. If the balance is made exactly, no current will be drawn from the source of the unknown voltage. A potentiometer can accordingly measure an emf by measuring it terminal voltage when no current is flowing from the emf.

A potentiometer circuit is shown in Fig. 172. The battery B furnishes a current in the circuit ABWDSA, so that the wire AD can be used as a potential divider (See Sect. 114). The variable potential needed to balance the unknown cell C is furnished between the end A and the sliding contact S. A galvanometer G is connected in series with C as shown. The positive terminal T of the cell will be higher in potential than A by an amount equal to the emf of C. The point S on the slide wire will be higher in potential than A by an amount equal to the potential drop along the slide wire from S to A. If S is chosen by trial so that no current flows through the galvanometer when T′ is touched to S, then the emf of C must be exactly balanced by an equal potential difference between S and A along the slide wire. If R is the resistance of the slide wire between S and A, then

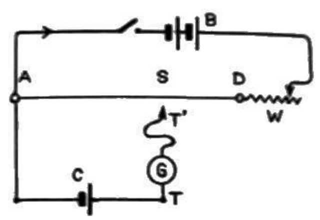

Fig. 172

$$E = IR \qquad (172\text{-}1)$$

Rather than to compute E from values of I and R, it is usually more accurate and convenient to eliminate I from the calculation by means of a second balancing with a standard cell C_1 of known emf E_1 substituted for C. If R_1 is the resistance between the new balance point S_1 and A, then

$$E_1 = IR_1 \qquad (172\text{-}2)$$

Since the current I is the same as before, it follows that

$$E/E_1 = R/R_1 \qquad (172\text{-}3)$$

For a uniform slide wire the resistance is proportional to the length involved so that

$$R/R_1 = AS/AS_1$$

A potentiometer circuit as shown in Fig. 172, is a network of resistances and emf's and hence all the laws for networks will apply.

If I is adjusted to begin with by means of W so that the length of wire required to balance a known emf is related numerically to the emf, the potentiometer may be made direct reading. For example, if the known emf is 1.08 volts, I could be adjusted until the length of slide-wire required to balance it was 108 cm. Then each cm along the wire would represent a difference in potential of .01 volt.

Compared to a moving-coil voltmeter for measuring potential differences, the potentiometer has a number of advantages. As mentioned above, it takes a very small current from the source being measured. The balancing principle makes it possible to measure a large emf with the sensitivity of a relative delicate meter. The accuracy of the measurement does not depend on the

accuracy of the scale of a moving-coil meter, because the meter is used only to indicate the absence of any noticeable current. The accuracy of the potentiometer measurement does depend on the precision of the standard cell and of the resistors used in the network, but accurate standard cells and resistors are easier to make and maintain than accurate moving-coil meters.

Chapter 18

THERMOELECTRIC EMF

180. When a closed circuit is formed from wires of two different metals A and B as indicated in Fig. 180, a current of electricity may flow even when there is no battery or other obvious source of emf. Such a current will flow when there is a difference in temperature between the two junctions H and C where the different metals are joined. Since there is no other emf in the circuit, and since the current flows only when there is a temperature difference, there must be an emf produced by the difference in temperature. This emf is referred to as a thermoelectric emf, and a combination of wires of two different metals in which such an emf may be generated is called a thermocouple.

Fig. 180

181. The Measurement of Temperature by Thermocouples. Since the emf in a thermocouple circuit depends on the difference in temperature between the two junctions, we can measure temperature differences by measuring the thermoelectric emf. This means that a thermocouple can be used as a thermometer. For example, the junction C shown in Fig. 180 could be put in a bath of ice water at 0°C, and the junction H could be placed in a furnace. The emf of the couple would then indicate the temperature of the furnace relative to the centigrade zero.

The emf in a thermocouple circuit can be measured by opening the circuit at any point and measuring the difference in potential between the two loose ends. For example, Fig. 181-1 shows a circuit like that of Fig. 180 except that it has been opened at the middle of the wire A, and a voltmeter has been connected between the loose ends X and Z to measure the difference in potential between these ends. The path from X to Z through the thermocouple is electrically equivalent to the path through a voltaic cell from one terminal to the other; it contains an emf E and an internal resistance. The internal resistance R_c is the resistance of the thermocouple wires from X to Z. The difference in potential V between X and Z may be referred to as the terminal voltage of the thermocouple (See Sect. 156). The equation

$$V = E - IR_c \tag{181-1}$$

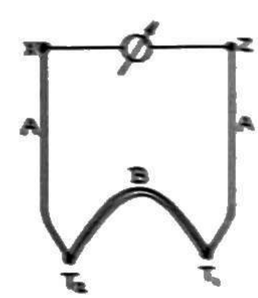

Fig. 181-1

applies when the emf of the couple drives a current I through the circuit, which now includes the voltmeter. If R_v is the resistance of the voltmeter, then

$$I = E/(R_v + R_c) \tag{181-2}$$

and

$$V = ER_v/(R_v + R_c). \tag{181-3}$$

This shows that the emf can be easily found from the observed reading of the voltmeter because

V is proportional to E. Voltmeters intended for use with a given thermocouple are often marked to indicate the temperature directly. The direct indications of such meters will be good only when used with a thermocouple made of the same materials and having the same resistance as the original couple.

If a potentiometer is used to measure the terminal voltage V, the current I in Eq. (181-1) will be negligible and the emf will be equal to the observed terminal voltage.

When a thermocouple circuit is opened to insert a meter to measure the emf, both of the loose ends must be kept at the same temperature to avoid introducing new thermal emfs that were not in the original circuit.

Thermoelectric emfs are relatively small and usually amount to a few millivolts for each hundred degrees difference in temperature. Pairs of metals to be used for measuring temperatures must first be calibrated by observing values of the emf corresponding to known temperatures, and plotting a graph of emf vs temperature.

In practice, pairs of metals are usually chosen which will give an emf that is approximately proportional to the difference in temperature between the two junctions. Graphs are given in Fig. 181-2 below for a number of pairs of metals, showing the emf of the couple plotted against the centigrade temperature T_2 of one junction when the other junction is maintained at 0°C. The graph for iron and copper is given in Fig. 181-3 to show that the emf is not proportional to the difference in temperature for all metals over unlimited ranges of temperature.

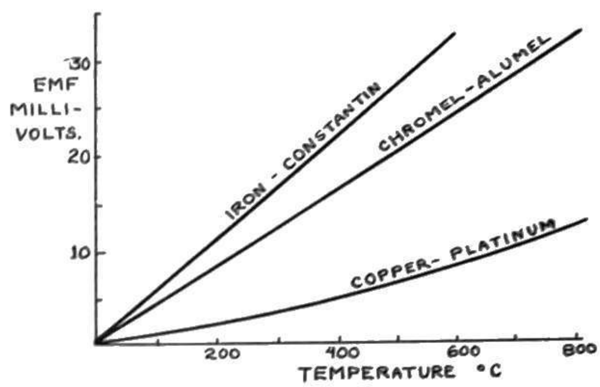

Fig. 181-2

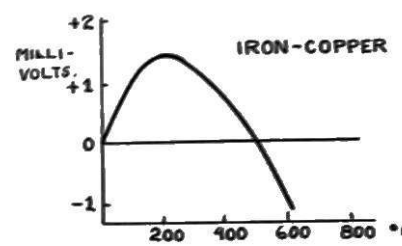

Fig. 181-3

Thermocouples are convenient because of the wide range of temperatures that can be measured by them. Some combinations are suitable for measuring temperatures as low as -253°C, and others can be used up to 1700°C. Another advantage of a thermocouple is that the temperature-sensitive junction can be inserted in inaccessible places with long wires extending to an indicating meter in a convenient location.

182. Thermopiles. A number of thermocouples may be connected in series and arranged so that all their hot junctions respond to the same temperature. If there are n couples and if the emf generated in each one is E, the total emf generated in the series combination will be nE. With this arrangement, a given temperature may be made to produce a larger and more easily measured emf. Such a combination of thermocouples is called a thermopile.

Thermopiles are frequently used to measure thermal radiation from hot bodies. To do this, the separate thermocouples may be arranged as shown in Fig. 182 so that the radiation is

concentrated on one set of junctions while the others are shielded from its effect. With such thermopiles, it has been found possible to detect the heat radiated from a lighted match several miles away, and measurements can be made on the heat radiated from stars.

Commercial devices called radiation pyrometers are used to measure the temperature of hot bodies by measuring the radiation emitted by the bodies. Most of these devices use a thermopile as the sensitive element.

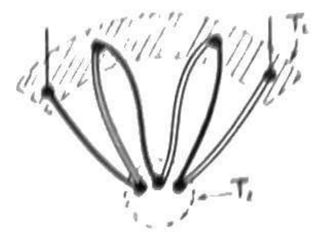

Fig. 182

183. Peltier Effect. Since the thermoelectric emf of a thermocouple is an effect due to temperature, it is reasonable to expect that it must be heat energy which furnishes the driving force. A simplified but incomplete explanation of the way in which thermal forces can drive electric charges may be made on the assumption that the free electrons in a metal behave like the molecules of a confined body of gas. As pointed out in Sect. 24, it appears that when metal atoms are combined in the solid state, the outer electrons of each atom may move around in the space between the atoms without being permanently attached to any one. The forces of electrostatic attraction demand that any small element of volume must contain practically all the electrons that belong to the atoms in that volume, even if individual electrons are not attached to individual atoms. In other words, the electrons are free to move around among the atoms, but they are not free to escape entirely from a piece of metal into empty space. In this way the surfaces of a piece of metal serve effectively as walls to confine the free electrons. Like the molecules of a confined gas, the free electrons appear to be moving about at random with a certain average amount of kinetic energy which increases as the temperature of the metal increases.

If two pieces of different kinds of metal such as iron and copper are joined together, there will be free electrons in each metal on both sides of the boundary at the junction. Although free electrons are retarded from escaping across a boundary into empty space, the electrons of one metal may diffuse into the other metal where the two metals are in contact. Because of the differences between any two metals, there will generally be a resultant diffusion of electrons in a given direction whenever those two metals are touched together. This diffusion takes place due to the thermal kinetic energy of the electrons andhence we may say that there is a thermal emf tending to drive electrons from the one metal into the other. If the thermal emf tends to drive electrons from B to A in Fig. 183-1 it would tend to drive positive charges in the opposite direction. Thus the emf would be said to act from A to B. As far as the electrical effect is concerned, this thermal emf is therefore equivalent to a voltaic cell inserted at the junction as indicated by E_{AB}

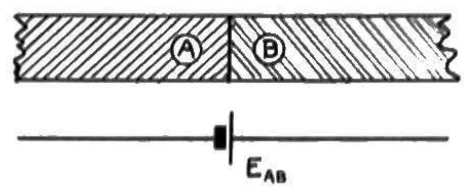

Fig. 183-1

The presence and direction of thermal emfs at the junctions between two wires can be demonstrated experimentally by sending a current through these junctions. For example, a current I may be sent through a piece of iron wire connected between two pieces of copper wire as shown in Fig. 183-2. The thermal emf at a copper-iron junction acts from the copper to the iron, so that the emfs at the junctions will be as indicated by equivalent voltaic cells in the figure. At the junction M, the current will flow in the direction of the emf, and the emf will expend heat energy to give the conventional positive charges more electrical energy as they pass through. At the junction N, the current is opposed to the emf, and the conventional positive charges will lose electric energy

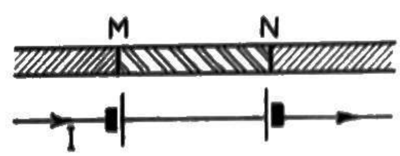

Fig. 183-2

which will be converted into heat energy. The existence of these emfs is verified by the fact that the junction at M is noticeably cooled when a current flows, as shown, while the junction at N is heated.

The heating or cooling of a thermocouple junction when a current passes through is referred to as a Peltier effect after the man who first discovered it, and the thermal emf at a junction is called a Peltier emf. The temperature change involved in the Peltier effect is so small that it is easily overlooked, particular since the independent heating effect of the current flowing through metallic resistance is usually much larger.

A thermocouple circuit composed of two metals always has two junctions as indicated in Fig. 183-3, where symbols for equivalent voltaic cells are drawn to indicate the Peltier emfs at the junctions. If both junctions are at the same temperature, the two Peltier emfs will be equal and opposite. However each Peltier emf increases with the temperature, so that if one junction is made hotter than the other, there will be a resultant emf around the circuit in the direction of the stronger emf.

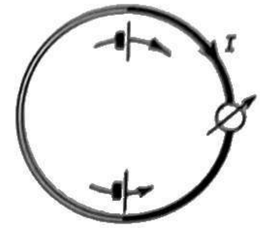

Fig. 183-3

When a current flows in a thermocouple, due to Peltier emfs, heat is absorbed at the hotter junction and converted into electrical energy. A smaller amount of heat is released at the colder junction because the emf is less, and the difference between the two amounts of heat is the net amount of heat converted into electrical energy. This process resembles heat engines in general, in that it absorbs an amount of heat at a higher temperature, discards a smaller amount of heat at a lower temperature, and converts the difference into another form of energy. A thermocouple thus accomplishes very simply the same thing that is accomplished in a power station by a steam turbine and a generator. It is only because the emfs in a thermocouple are so small that it is not of much value in supplying electric power for the sake of the power itself. It is of interest to note that if a current is sent through a thermocouple in a direction opposite to the emf of the couple, heat will be absorbed at the cooler junction and emitted at the hotter junction. A thermocouple can therefore act as a refrigerator in which electric power is used to convey heat away from a spot that is already cooler than its surroundings. Here again the emfs involved are too small for practical application as a power handling device.

184. The Thompson Emf. In addition to the emfs appearing at the junctions of a thermocouple, there are similar emfs generated along the wires themselves due to the drop in temperature from the hot junction to the cold junction. This emf depends on the diffusion of the electrons from one point in a metal to another point in the same metal where the temperature is lower. Thus in a part of a wire as illustrated in Fig. 184, electrons in the hot region will diffuse toward the cold region making the cold region relatively negative. This action amounts to an emf in the conventional sense from the cold to the hot region, which would have the same electrical effect as a voltaic cell inserted in the path as indicated. Such a thermal emf in a uniform piece of wire is known as a Thompson emf. If a current flows through a wire along which there is a temperature variation, the Thompson emf will produce a heating or cooling effect depending on the direction of the current. Like the Peltier effect, this effect is small and cannot be detected without careful measurement.

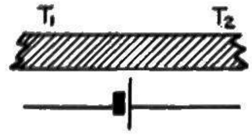

Fig. 184

The direction of the Thompson emf in many metals is found to be as would be expected from the simple theory of electron diffusion. In other metals, an observed Thompson emf in the opposite direction indicates that an increased temperature must have other effects in the metal which outweigh the effect of an increased thermal velocity of the diffusing electrons.

Although the Thompson emfs in a thermocouple contribute to the resultant emf, the natures of the Thompson emfs and the Peltier emfs are such that their resultant contribution to the total emf depends only on the difference in temperature between the two junction points, and not on the distribution of temperature along the wires.

Chapter 18A

CONDUCTION IN GASES

186. The electrical properties of any conductor may be described by a graph which shows how much difference in potential V will be required for any given amount of current I. For metallic conductors at a constant temperature, V is proportional to I and the behavior can be described simply by giving the value of the fixed ratio of V/I, which is called the resistance of the conductor. For gaseous conductors, the ratio of V to I is different for different values of V or I, and the behavior of these conductors can not be conveniently described except by graphs of V vs I. In order to illustrate the behavior of gas as a conductor of electricity let us consider a body of gas confined in a glass tube so that it serves as a path to complete a circuit containing an emf E and a variable resistance R, as shown in Fig. 186. The current I is led into the body of gas through a metal electrode M and out at the other end through a similar electrode N. The current I passing through the cross section at any point will be equal to the amount of positive charge passing the point per unit time in the direction of the current, plus the amount of negative charge passing the same point in the opposite direction.

There are many different types of behavior exhibited by gaseous conductors depending on the pressure of the gas, the nature and temperature of the metal electrodes, etc. Of all these we shall consider three particular types.

When gaseous conducting paths are connected to emfs with other conducting paths, usual laws for current and voltage in connected paths apply as previously given in Sects. 104 and 105.

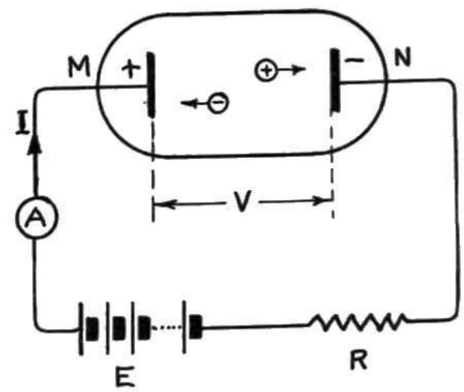

Fig. 186

187. Conduction Maintained by an External Ionizing Agent. If a low voltage of the order of a few volts is applied between the electrodes, the current which flows will be merely the small current due to the ions which are formed by any existing independent ionizing agent such as cosmic rays or x-rays which strike the gas. This type of discharge depends entirely on the external ionizing agent to furnish the ions which make the gas conducting. If the voltage V applied to a body of gas between two electrodes is increased from zero, the current I will at first increase. This occurs because the increasing voltage pulls more and more of the ions formed in the gas to the electrodes before they have a chance to recombine. This behavior is illustrated in Fig. 187 which shows that I at first increases as V increases. When a voltage V_1 is reached such that all the ions are pulled to the electrodes as fast as they are formed by the external ionizing agent, then a further increase in the voltage to a value V_2 does not produce any change in the current. This constant value of the current is determined by the rate at which ion pairs formed by the ionizing agent.

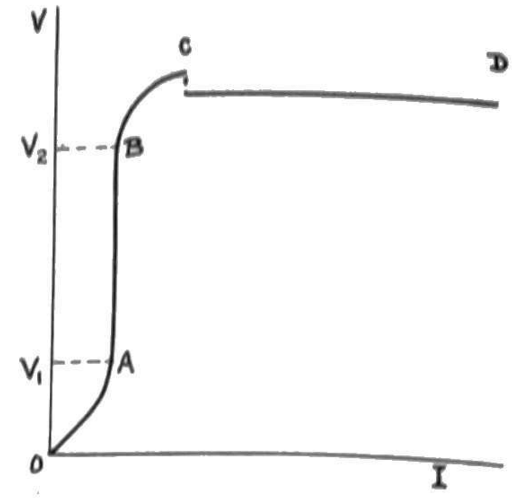

Fig. 187

188. Glow Discharge. If the voltage applied to a gaseous conductor as described in part (b) above is increased enough, the ions present in the discharge will acquire enough energy between successive collisions so that they will break up the neutral molecules which they strike. Each molecule broken up forms an additional pair of ions, and each of these as well as the original ion may in turn proceed to break up still more. Thus after a critical voltage is reached, an avalanche of ions will be formed. Depending on the pressure of the gas and the geometry of the discharge tube, it may turn out for a considerable range of current values that any amount of current can flow without any change in potential difference between the electrodes. This may not seem unreasonable in view of the fact that once a critical voltage is reached the comulative process of ionization makes available as many ions as can be used. Under this condition the amount of current will be controlled by the rest of the circuit. Once such a discharge gets started, it is of course no longer dependent upon the external ionizing agent which furnished the initial ions.

Discharge tubes operated in the region where the voltage is constant for a wide range of current values find application as voltage control tubes in various electronic devices. Such tubes tend to maintain a fixed difference in potential between any two terminals connected to the electrodes regardless of changes in the supply voltage or changes in the resistance of the rest of the circuit.

The glow discharge derives its name from the fact that light is emitted from the gas as it conducts the current. The production of light by a body of gas always accompanies any extensive ionization by collision in a body of gas.

The gas pressure in glow discharge tubes is usually from about .0001 to .1 of an atmosphere.

189. High Pressure Arcs. A gas discharge can be made to take place through a gas at atmospheric pressure if the two electrodes are first touched together and then separated slightly. The maintenance of such an arc depends upon the electrodes being heated to a high temperature by the discharge, and the discharge is carried partly by ions of the gas, partly by ions of the vaporized electrode.

An arc behaves as if an increase in current produces more ions so that less voltage is required for the larger current. The curve of V vs I therefore has a downward slope with smaller values of V corresponding to larger values of I. An ordinary carbon arc is an example of this type of discharge.

The tip of the positive carbon becomes white hot in a carbon arc, and it is the chief source of light in such an arc. Some light is also emitted for the gas discharge between the carbon electrodes. Because of the fact that the voltage across an arc decreases with the current, a resistance must be used in series with an arc to give stable operation.

Chapter 19

MAGNETIC FORCES AND FIELDS

190. Magnets were known to exist for centuries before any connection between magnetism and electricity was recognized. Magnetism was accordingly regarded as an independent phenomena. However it now appears that magnetic forces come from the motion of electric charges, so that magnetism can be treated as a part of the more general phenomena of electricity. For example, it appears that the forces between two magnets are forces between electrons which are spinning around inside the atoms in the magnets. The magnetic forces between two electric currents are forces that act between the moving charges which constitute the currents. These magnetic forces between moving charges act only when the charges are moving, and they act in addition to the electrostatic forces about which we have already studied.

In the following study of magnetism we will first review the more obvious properties of magnets as such. After that, we will develop the theory of magnetism from the electrical point of view. In Chap. 27 we will return to a further consideration of magnets to show how their peculiar properties can be attributed to the motion of the charges within the magnets.

191. Magnets and Magnetic Poles. Magnets are familiar to every one as bodies of metal which will attract pieces of iron or steel. Thus if a horseshoe magnet is dipped in a box of iron filings, the filings will hang from the tips of the horseshoe as shown in Fig. 191-1. A bar shaped magnet will attract the filings at its ends as shown in Fig. 191-2. The ends of a magnet from which the attractive forces seem to originate are called poles.

A magnet that is mounted so it can turn freely will ordinarily turn until one particular pole points to the north. This behavior shows that there is a difference between the two poles of the magnet. The pole which is drawn to the north is called a north or positive pole. The other pole is called a south, or negative pole. A magnetic compass consists of a magnetized needle mounted horizontally to swing freely about a vertical axis. Such compasses have long been used as direction finders in navigation.

In general, the poles of a magnet will attract neutral pieces of iron or steel, and the north pole of one magnet will attract the south pole of another magnet. However a pole will exert a repelling force on another pole of the same kind.

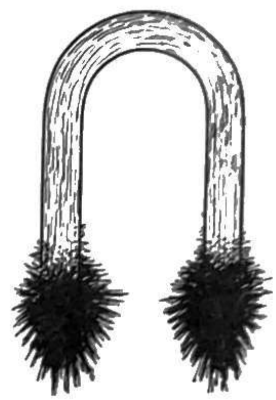

Fig. 191-1

192. Magnetic Fields As Observed by a Compass. A magnetic field is a directive influence in a region which can be detected by its effect on a compass needle. For example, everyone is familiar with the earth's magnetic field as the influence which makes compass needles point to the north, as mentioned above. Magnetic fields other than the earth's field may exist, and we shall have more to say later about the ways in which magnetic fields can be produced and measured. For the present we will merely note than any magnetic field can be detected by a compass, and that the direction of the magnetic field at any point is the direction that the north pole of a compass will point if placed there.

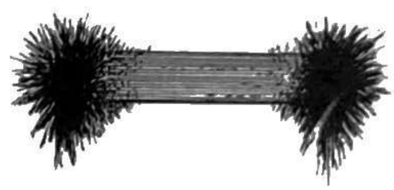

Fig. 191-2

193. Fields due to Permanent Magnets. If a compass needle is placed near either pole of a relative large magnet, the needle will in general point away from the north pole of the large

magnet and toward the south pole, as indicated by the short arrows in Fig. 193-1. This shows that a large magnet produces a field of its own that is strong enough to obscure the earth's field at points near the magnet. A complete map of the field of a magnet can be made by placing a compass at various points around the magnet, and indicating the direction at each point by an arrow as was done in Fig. 193-1.

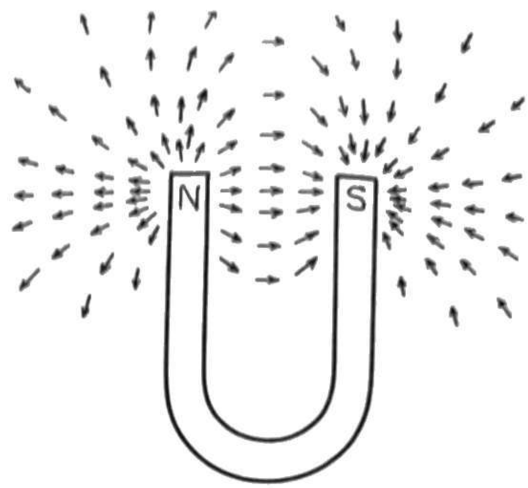

Fig. 193-1

A map of the magnetic field at all points around a magnet can also be made by using iron filings sprinkled over a sheet of glass or paper placed horizontally over the magnet. Each small piece of filing is magnetized by the field and then behaves like a small compass needle. These minature compass needles line up end to end with the north pole of one holding to the south pole of the next, indicating the direction of the field at all points as shown in Fig. 193-2.

Whether a magnetic field is mapped by using a compass needle or by using iron filings, the configuration of the field can be represented by drawing continuous lines to show the direction of the field at all points. These lines are called magnetic lines of force.

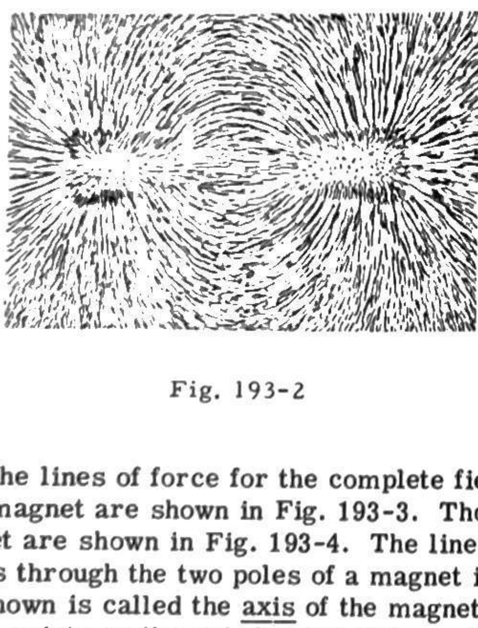

Fig. 193-2

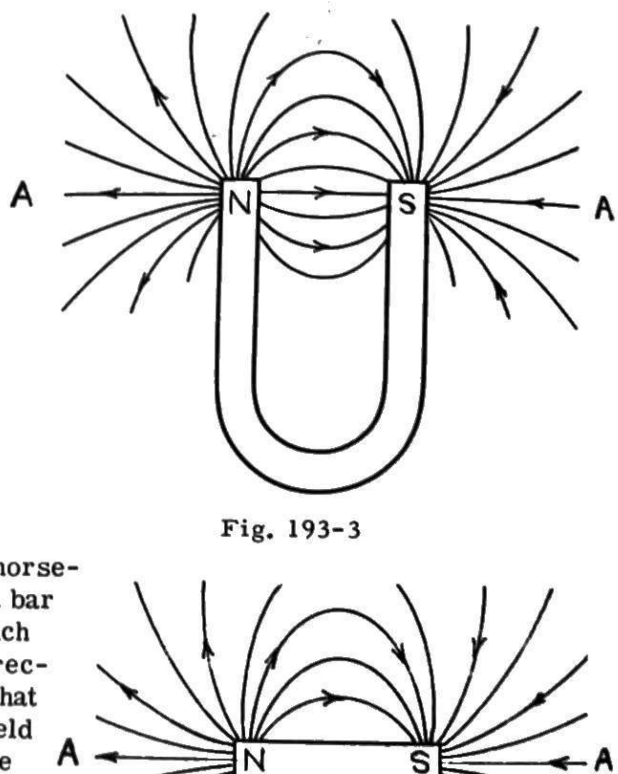

Fig. 193-3

The lines of force for the complete field of a horseshoe magnet are shown in Fig. 193-3. Those for a bar magnet are shown in Fig. 193-4. The line AA which passes through the two poles of a magnet in the direction shown is called the axis of the magnet. Note that for all points on the axis beyond either pole, the field is in the direction of the axis. For all points on the perpendicular bisector of the axis, the field is parallel to the axis and opposite in direction. It would be possible to make a magnet with a field that was not as symmetrical as those shown in the above figures, but horseshoe and bar magnets are generally magnetized to give fields as shown.

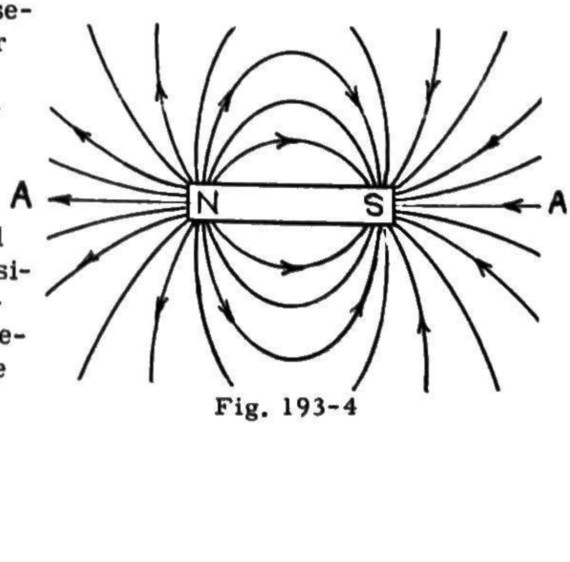

Fig. 193-4

194. The Magnetic Field of the Earth. As stated above, there is a natural magnetic field associated with the earth which acts in a northerly direction over most of the earth's surface. Magnetic compasses are ordinarily used in a horizontal plane, but the earth's field is not everywhere horizontal. At points where earth's field is not horizontal, a compass utilizes only the horizontal component of the field (See Sect. 211).

To find the angle of the earth's field above or below the horizontal, a so-called dipping needle can be used. A dipping needle is a magnetized needle mounted so it can swing in a vertical plane up or down from the horizontal position which a compass needle would have at that point. The angle between the horizontal and the direction of the earth's field at any place is called the angle of dip, or inclination.

The general configuration of the earth's field is illustrated in Fig. 194. This figure shows lines of force relative to a cross-section of the earth taken through the axis of rotation. Note that for points in the northern hemisphere, the field is directed downward from the horizontal by an angle of dip which in general increases with the latitude. At the equator, the field is approximately horizontal. In the southern hemisphere the field is directed upward from the horizontal.

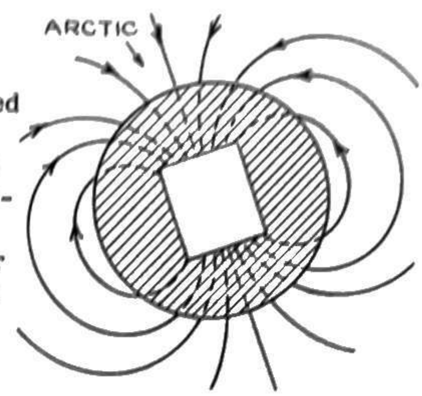

Fig. 194

As indicated in Fig. 194, the field of the earth resembles that of a bar magnet with the north magnetic pole in the southern hemisphere, and the south magnetic pole in the northern hemisphere. This apparent inconsistency arises because the north pole of a bar magnet is defined as the pole which seeks the northward direction in the earth's field.

The specific location of the earth's magnetic pole in the northern hemisphere is the spot on the earth's surface where the field is directed vertically down into the earth. At present it is located approximately at a latitude 70° north and longitude 96° west. It thus does not coincide exactly with the north geographic pole. It follows that compass needles will not in general point exactly north along the geographic meredians. The angle between the true geographic north and the magnetic north as indicated by a compass needle at any place is referred to as the magnetic declination for that place. Angles of declination in the United States range from about 12° west for places on the Atlantic coast to about 18° east for places on the Pacific coast. Detailed data on both the declination and the inclination of the earth's field at various places are available in handbooks.

Although the configuration of the earth's field is in general as described above, the field is not completely symmetrical nor is it absolutely constant. Irregular minor variations occur from place to place, and from time to time. The changes observed with respect to time include erratic changes which vary from day to day, and progressive changes which persist somewhat uniformly over longer periods of time. The erratic type of variation appears to be correlated with the existence of sun spots.

Chapter 20

MAGNETIC FORCES AND FIELDS (continued)

200. Magnetic Forces on Currents. Motor Effect. One of the most important features of a magnetic field is that it can exert a sidewise force on a wire carrying an electric current. This sidewise force can be demonstrated by suspending a vertical wire in the field of a horseshoe magnet as shown in Fig. 200-1. When a current I is passed through the wire as indicated, the wire will swing away from the magnet, showing the existance of a force F. The sidewise force on a current flowing through a magnetic field is the fundamental phenomena involved in the rotation of an electric motor and hence this effect is often referred to as the "motor effect." The amount of force which a magnetic field can exert on a given current can be used as a measure of the magnitude of the field, as will be explained in Sect. 201 below. The field defined in this way will then be a vector quantity having magnitude as well as direction, and we will represent it by the symbol B.

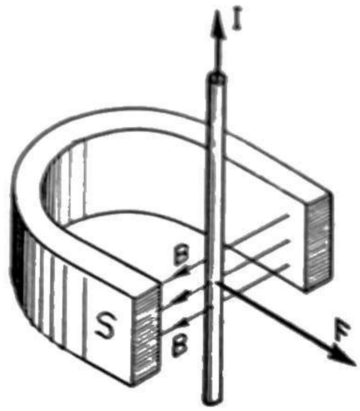

Fig. 200-1

The force of a magnetic field B on a current I is always perpendicular to both I and B. In other words, the force is perpendicular to the plane which contains the vectors I and B as shown in Fig. 200-2. The relative directions between the three vectors can be described if we write the three vectors in the easily remembered order FIB. The three vectors will then be related by the right hand screw rule like the three axes of a right handed set of coordinate axes. Thus in Fig. 200-2, the positive direction of the first named vector F perpendicular to the I-B plane is found by rotating the tip of the second vector I through an angle ϕ in the I-B plane toward the tip of the third vector B and taking the direction in which this rotation would advance a right-hand screw along F as indicated. The reader may find it helpful to visualize these directional relationships by labeling the thumb, the first finger, and the second finger of the right hand with the symbols FIB in that order, and then holding the thumb and fingers to point in three mutually perpendicular directions as illustrated in Fig. 200-3.

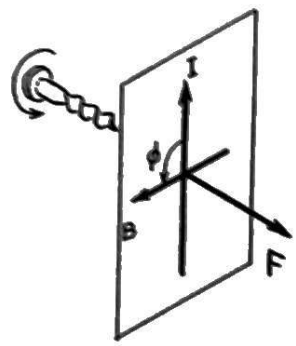

Fig. 200-2

As stated above, the force on a wire carrying a current I in a magnetic field B will always be perpendicular to the geometrical plane formed by the vectors for I and B. This is true even if the wire is placed in the field without being perpendicular to the field. In general, the angle ϕ between the current and the field may have any value between 0° and 180°, but the two vectors will still form a plane, and the force on the wire will be perpendicular to this plane. For given values of I and B, the

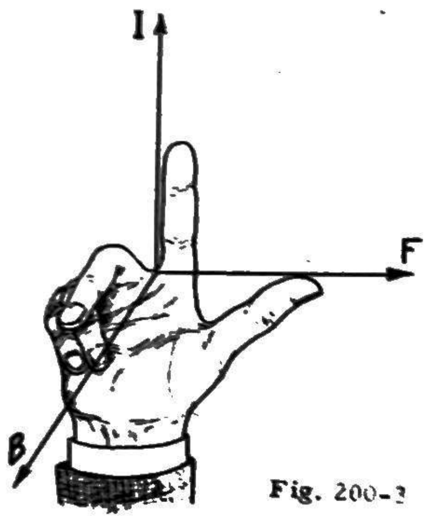

Fig. 200-3

magnitude of the force will depend on the angle ϕ between I and B, and it will have a maximum value F_m when this angle ϕ is 90°. For any other value of ϕ, the force F will be less than the maximum force according to the equation

$$F = F_m \sin \phi \qquad (200)$$

This means that the force approaches zero as the angle ϕ approaches zero. The force on the current is found to be directly proportional to the current I and also to the length L of the piece of wire in which the current flows, other things being equal.

As examples of the above relationships, note Fig. 200-4, where three equal currents I, I_1, and I_2 all lie in the same I-B plane. All three will experience a force F in the same direction perpendicular to the I-B plane, but the magnitude of the force on I will be greatest since the angle between I and B in the I-B plane is 90°. In Fig. 200-5, all three currents will experience a downward force perpendicular to the I-B plane, but the force on I_4 will be greatest because it makes an angle ϕ of 90° with B.

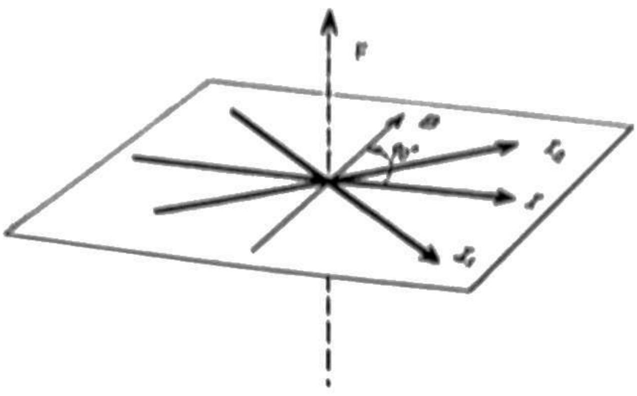

Fig. 200-4

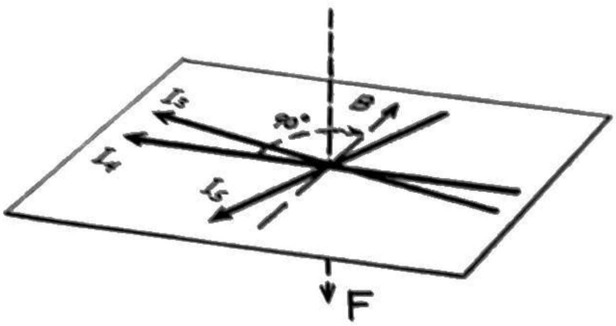

Fig. 200-5

201. Magnetic Flux Density. The maximum amount of force per unit current per unit length of wire is called the flux density of the field. According to this definition, the flux density B at any point can be computed by the equation

$$B = \frac{F_m}{IL} \qquad (201\text{-}1)$$

where F_m is the maximum force. As explained in the preceding section, I must be perpendicular to B to give this maximum force. Magnetic flux density B is also referred to as magnetic induction.

To illustrate the meaning of flux density, consider a straight wire 2 meters long placed perpendicularly in the earth's field to measure the flux density of the field. If a sidewise of .00025 newtons is observed to act on the wire when a current of 5 amperes flows, the flux density B of the earth's field would be given by

$$B = \frac{.00025 \text{ newtons}}{5 \text{ amp} \times 2 \text{ meter}}$$

or

$$B = 2.5 \times 10^{-5} \text{ newt/amp-meter} \qquad (201\text{-}2)$$

Since the observed force is proportional to I and L for a given uniform field, the same ratio of

F_m/IL would have been obtained for any value of I or L which might have been used. If B is not the same at all points along the wire, then a very short wire should be used to give the value of B at any point.

Once the value of B is determined at a point by measuring the force on a known current, the maximum force that will act on any other current can be computed by Eq. 201-1. For this computation, the equation may be written

$$F_m = BIL. \tag{201-3}$$

If the angle ϕ between I and B is not a right angle, the force according to Eq. (201-1) will then be given by the equation

$$F = BIL \sin\phi . \tag{201-4}$$

202. Units of Flux Density. As may be seen in Eq. (201-2), the mks practical unit of B is that amount of flux density that will exert a force of one newton on a current of one ampere in a wire 1 meter long. This unit is well described by the name "newton per ampere-meter", which will be used when convenient. Magnetic flux density may be represented by drawing lines of force as will be explained in Sect. 222, and hence this mks unit is also referred to as "lines of flux" per m^2 or "webers" per m^2.

The gauss is a smaller unit of flux density, related to the newton per ampere-meter by the equation

$$10^4 \text{ gauss} = 1 \text{ newton per amp-meter} \tag{202-1}$$

Although the size of the gauss was originally determined by its use in the cgs system, it can be used in the mks system as a convenient fraction of the mks unit. For example, the flux density of the earth's field, which is of the order of .2 gauss, would be less conveniently expressed as 2×10^{-5} newt/amp m.

203. Direction of a Magnetic Field as Indicated by a Coil. A coil carrying a current in a magnetic field will tend to assume one particular position with respect to the field. For example, a rectangular turn of wire suspended between the poles of a magnet will tend to take the position shown in Fig. 203-1. In this equilibrium position, the plane of the coil is perpendicular to the field, with a right-hand screw relationship existing between the direction of the current around the coil and the direction of the field through the coil. That is, the direction of the current around the inclosed lines of flux is the direction in which a right-hand screw would rotate to advance along the lines of magnetic force.

The reason for this equilibrium position can be seen in Fig. 203-2. This shows a cross-section through the coil of Fig. 203-1 looking down from above. It follows from Sect. 200 that the horizontal field B will exert outward forces FF on the sides of the coil. In Fig. 203-2, the coil is shown

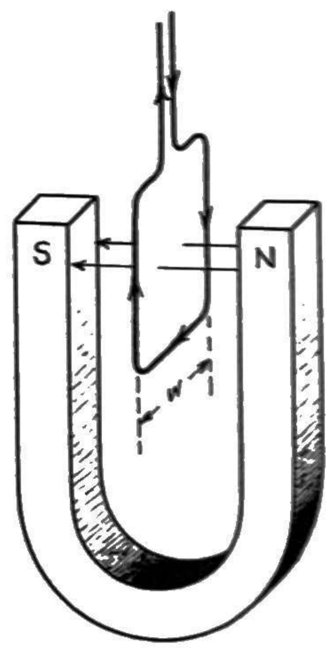

Fig. 203-1

turned away from its equilibrium position PQ by an angle θ. It can be seen that the forces FF will tend to turn the coil back to its equilibrium position where the plane of the coil will be perpendicular to the field.

It follows from above that the observed force on moving charges such as those in a coil of wire can be used to determine the direction of an unknown field. This direction as observed by a current in a loop of wire will be the same as the direction indicated by a compass needle at all points which are accessible for actual observation. To cover hypothetical cases where the two directions might be different, the direction of B is by definition taken to be the direction as indicated by moving charges. This means that both the magnitude and the direction of the flux density B are ultimately defined in terms of the force it will exert on a moving charge, and without any necessary reference to magnets as such.

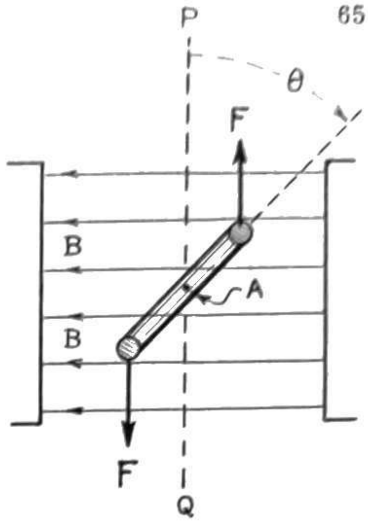

Fig. 203-2

204. Torque on a Flat Coil in a Uniform Magnetic Field. For a rectangular coil with vertical sides placed in a horizontal field as shown in Fig. 203-1, the torque about a vertical axis can be computed easily. This torque will depend upon the angular position of the coil relative to its equilibrium position, and we shall compute it for an angular position θ as shown in Fig. 203-2. Let L represent the length of a verical side of the coil and I the current in the coil. The horizontal force F on either side of the coil will then be given by the equation.

$$F = BIL \tag{204-1}$$

The lever arm of either force about the axis A in Fig. 203-2 will be equal to half of the width of the coil times the sine of the angle θ. The torque L_1 of either force will therefore be given by

$$L_1 = BIL(w/2) \sin\theta \tag{204-2}$$

Both torques act around the axis in the same direction, and hence the total torque L will be equal to $2L_1$. Thus we may write

$$L = BILw \sin\theta. \tag{204-3}$$

This equation gives the torque on a coil having only one turn of wire. For a coil having N turns, the torque will be N times as great.

The product Lw in Eq. (204-3) is the area A of the coil. Thus the equation may also be written

$$L = BIA \sin\theta. \tag{204-4}$$

205. Torque on any Flat Coil in a Magnetic Field. The equations of the preceding section were derived for a rectangular coil with vertical sides in a horizontal field. By considering any flat coil to be equivalent to an infinitely large number of very small rectangular coils, it can be shown that Eq. (204-4) is a general equation that will apply to any flat coil. In the general application of this equation, θ is the angular displacement of the plane of the coil away from the equilibrium position. The torque on the coil will be in the direction which tends to decrease this angular displacement θ.

206. Motion of a Charge in a Magnetic Field. The magnetic field which exerts a side-wise force on a current-carrying wire will also exert a similar force on any moving charge. For example let us suppose that a positively charged particle is shot like a bullet horizontally to the east through a magnetic field which is directed upward. The moving charge is equivalent to a current directed east. It will accordingly experience a deflecting force to the south just as an equivalent current would experience a sidewise force to the south.

In general, the direction of the force on any charge moving in a field can always be found by considering the charge as an equivalent current. A negative charge moving with a velocity v is equivalent to a positive current moving in the opposite direction. Hence a negative charge moving east through an upward field will experience a northward deflecting force like a current moving to the west.

The magnitude of the sidewise force on a charge q moving with a velocity v in a field B is given by the equation

$$F = qvB \sin \phi \tag{206-1}$$

where ϕ is the angle between B and v. This equation is the same as Eq. (201-4) except that the product qv appears instead of the product IL. This is consistent because there must be a body of charge moving with a certain velocity in a wire if there is a current along the wire. (See Appendix B)

Since the force of a magnetic field on a freely moving charge is always perpendicular to its velocity, it will deflect the charge sidewise without changing its speed. This continuous sidewise deflection with a constant speed gives uniform motion in a spiral path, with the sidewise force of the field acting as the centripetal force. If the charge is originally moving perpendicular to the field, the path will be a closed circle in a plane perpendicular to the field. For example, a positively charged particle initially moving to the east through an upward magnetic field will swing around to the south in a horizontal circular path. When it is moving south, it will be deflected west, and so on. Thus it will move in a horizontal circle in a clockwise direction as viewed from above.

The curvature of the path of a charge moving in a magnetic field depends upon the flux density of the field, and on the nature and the speed of the particle. If a particle having a charge q and a mass m is moving with a velocity v perpendicular to a field B, the centripetal force will be qvB. This must be equal to the product of the mass times the centripetal acceleration v^2/r, where r is the radius of the resulting circular path. Hence we may write

$$m \frac{v^2}{r} = qvB \tag{206-2}$$

or

$$r = mv/qB \tag{206-3}$$

If a charged particle has a velocity which is not perpendicular to the magnetic field, the curvature of the resulting spiral path can be found by resolving the motion into two components. The component of the motion perpendicular to the field will be a circular motion obeying Eq. (206-3). The component of the motion parallel to the field will be unaffected by the field since there is no force on a charge moving parallel to the field. Thus there will be a circular motion perpendicular to the field superimposed on a constant velocity in a direction parallel to the field. The resulting motion will be along a cylindrical spiral with the axis of the spiral parallel to the field.

Chapter 21

VECTOR COMBINATIONS OF MAGNETIC FIELDS

210. Resultant of Several Magnetic Fields. If two or more magnetic fields are superimposed at the same point, the component fields cannot be observed as such. Any measurement of magnitude or direction will give only one observed value for the field at a given point. If several fields are acting simultaneously, the observed field is found to be the resultant of the several fields, added as vectors. For example, suppose the westward field B_m of a magnet is superimposed on the northward field B_e of the earth at a point P, as shown in Fig. 210. A compass needle placed at P will then point in the direction of the resultant field B, and will not detect the separate components.

It follows from above that the magnitudes of two superimposed fields can be compared if their directions are known and if the direction of their resultant is observed by a compass. Thus if the directions of B_e and B_m are known in Fig. 210, and if the direction of the resultant B is observed, the ratio of B_m to B_e will be given by the equation

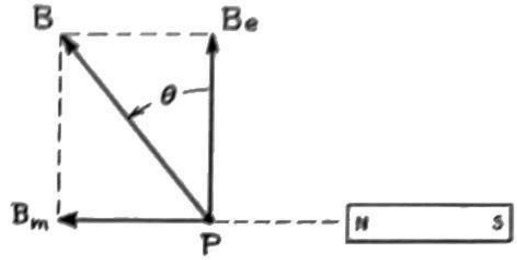

Fig. 210

$$B_m/B_e = \tan\theta . \qquad (210)$$

211. Resolution of a Magnetic Field into Components. It follows from the vector nature of flux density that one given field can be resolved into two components. For example, consider the earth's field B at a point where it is directed down into the earth, making an angle of dip ϕ with the horizontal as shown in Fig. 211. B will then have a vertical component B_v equal to B sin ϕ and a horizontal component B_H equal to B cos ϕ. The effect of B in any situation will thus be the same as the effect that would be produced by two independent fields B_H and B_v acting simultaneously.

In using a compass, it must be remembered that a vertical field has no effect in rotating a horizontal needle in a horizontal plane. Hence the effect of any field on a horizontal compass will be the effect of its horizontal component only. For this reason the magnitude of the earth's field is generally given in terms of its horizontal component. The flux density of the horizontal component of the earth's field in the United States varies from about .27 gauss in the southern part to about .16 gauss in the northern part.

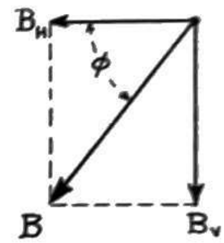

Fig. 211

* The earth's field may be expressed either in terms of its flux density B, or in terms of a corresponding magnetic intensity H. (See footnote, p. 87). When it is expressed in terms of the latter, it is usually given in terms of a unit called the "oersted". Any such data expressed in oersteds can easily be expressed in terms of the corresponding flux density B by remembering that the flux density in gauss for any point in empty space is numerically equal to the field intensity in oersteds. For example, if the earth's field at a given point is stated to have an intensity of .18 oersteds, it will also have a flux density of .18 gauss in the same direction.

Chapter 22

MAGNETIC FLUX

220. Lines of Magnetic Flux Density. The magnetic flux density B in a given region can be represented by continuous geometrical lines drawn so that they are everywhere parallel to the direction of B. To represent the magnitude as well as direction of B at all points, the lines are drawn close enough together so that the number of lines through a unit perpendicular area is equal to the number of units in the value of B at that point. These lines are referred to as lines of flux. Illustrative lines of flux are drawn in Fig. 220-1 to show the direction of B at points near the north poles of two bar magnets. These magnets are the same size but have different strengths. Note that the patterns of the two fields are similar, but the lines from the second magnet are spaced closer together to indicate larger values of B in the region near its pole. For either magnet, the crowding of the lines increases as we approach the pole, just as the value of B increases.

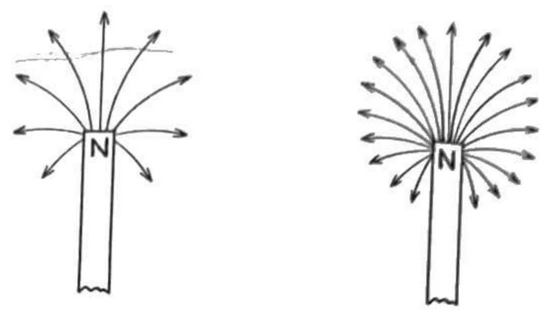

Fig. 220-1

It will also be noted that the pattern of the lines of flux is just like the pattern of lines of force from the north pole of a magnet (See Sect. 193). This of course must be true because the direction of a magnetic field as determined by a compass is the same as the direction of the flux density B as determined by a current carrying coil (See Sect. 203).

The total number of lines of flux that must be drawn through an area A to represent the flux density B in that region will depend on the units in which B and A are expressed. For example, if B has a value of 50 mks units directed vertically up as indicated in Fig. 220-2, it will require 50 vertical lines per unit area through the horizontal surface APQD. If APQD has an area of four meters, it will therefore require 200 lines through the whole area. If we wish to find the number of lines passing through an area which is not perpendicular to B, then we multiply the numerical value of B by the perpendicular component of the area as seen looking along the direction of B.

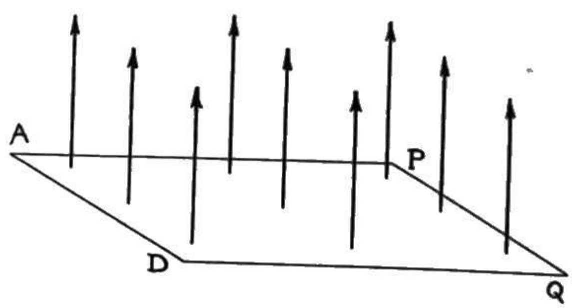

Fig. 220-2

221. The Definition of Magnetic Flux. If lines of flux are drawn through a given area so that the number of lines per unit area represents B, then the total number of lines through the whole area represents another quantity of great importance. This quantity is referred to as the magnetic flux through the area involved. Represented by the symbol Φ, the magnetic flux through an area is defined by the equation

$$\Phi = BA_{\perp} \qquad (221)$$

where $A_{\perp}$ is the perpendicular component of the area as seen looking along the lines B. For an example, let us refer again to Fig. 220-2. If B is equal to 50 mks units as represented by 50 lines of flux per unit area, then Φ = (50 units of B) x (4 units of area) = 200 units of flux. These 200 units of magnetic flux correspond to the 200 lines which pass through the area.

222. Units of Magnetic Flux. According to the definition $\phi = BA_\perp$, the amount of magnetic flux through an area of 1 square meter at a point where B is 1 practical unit will be given by the equation

$$\phi = (1 \text{ mks unit of B}) \times 1 \text{ m}^2) \tag{222-1}$$

This amount of flux is taken as the mks unit of flux, and is called a weber. Hence we may write

$$1 \text{ weber} = (1 \text{ mks unit of B}) \times (1 \text{ m}^2) \tag{222-2}$$

This equation may also be written

$$\boxed{(1 \text{ mks unit of B})} = 1 \frac{\text{weber}}{\text{m}^2} \tag{222-3}$$

Thus it is that the mks unit of flux density is commonly referred to as a weber/m². It will be recalled that the practical unit of B can also be written as 1 newton per ampere-meter. In other words, it follows from the definition of a weber that

$$1 \frac{\text{newt}}{\text{amp-m}} = 1 \frac{\text{weber}}{\text{m}^2} \tag{222-4}$$

The weber, abbreviated web, is an inconveniently large unit for many calculations, and in such cases the microweber may be used where

$$1 \text{ microweb} = 10^{-6} \text{ web.} \tag{222-5}$$

Because the number of lines of flux through an area represents the number of units of flux, the word "line" is often used as a universal synonym for a unit of flux. The exact meaning of the word will depend on the unit which is implied. Thus a "line" of flux may mean either a weber or a microweber, depending on which unit is used to express the flux.

The association of units of flux with lines of flux permits a picture of the relationship between a microweber and a weber. Thus if we think of a microweber as one thin line of flux, we may think of a weber as a thicker line of flux containing 10^6 of the thinner lines, just as a large rope is made of many thinner strands. (See Fig. 222).

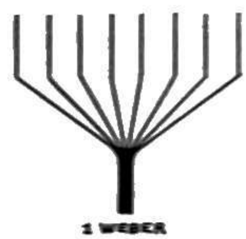

Fig. 222

Chapter 23

MAGNETIC FIELDS DUE TO CURRENTS

230. An electric current always produces a magnetic field in the surrounding space. For example, a current flowing in a straight wire will produce lines of force forming closed circles about the current as shown in Fig. 230-1. The existence of this field can be verified by holding a compass needle at various points around the wire. The magnetic lines of force around a wire can also be demonstrated by using iron filings on a horizontal plate as shown in Fig. 230-2. There the current carrying wire passes vertically through a hole in the plate. The direction of the lines of force around the wire in any case is the direction in which a right-hand screw would have to rotate to advance in the direction of the current, as indicated in the above figures.

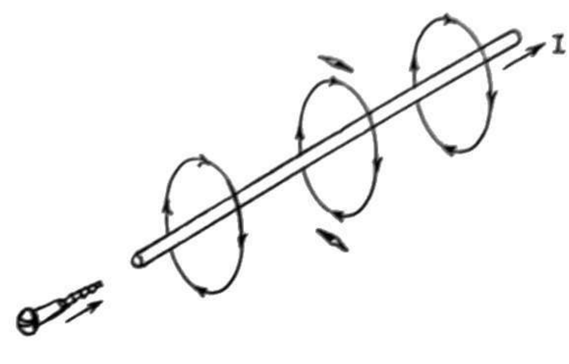

Fig. 230-1

When a long current-carrying wire produces a field B at any point P, each short segment of the wire contributes a part to the total field at P. The contribution dB of any particular segment having a length dL depends on a number of factors: It is inversely proportional to the square of the distance r from the segment to the point P. It is directly proportional to the length dL of the segment and to the amount of current I flowing in the segment. It is directly proportional to the sine of the angle θ between the direction of the current and the direction of a line drawn from the segment to the point P, as shown in Fig. 230-3. Taking all of these factors into account, we may write

$$dB = \frac{\mu_0}{4\pi} \frac{idL \sin\theta}{r^2} \qquad (230\text{-}1)$$

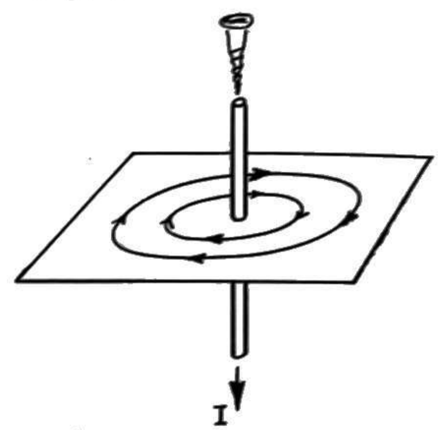

Fig. 230-2

where μ_0 is a factor of proportionality that can be determined by experiment.

Any segment to which Eq. (230) can be applied directly must be short enough so that θ and r are the same for all parts of the segment. Ordinarily this requires that any actual wire must be divided into a very large number of infinitely small segments. In that case the computation cannot be made without using calculus. Segments of appreciably length can be handled directly by Eq. (230-2) without calculus only if the wire is arranged so that r and θ are the same for all parts of the segment.

The lines of force representing the contribution of dL to the total field will be circles perpendicular to and centered on a straight line extended along dL. The direction of these circles around the extension of dL follows the right-hand screw rule as may be seen by comparing Fig. 230-3 to Fig. 230-2. In other words, the field at any point due to a short segment of wire (Fig. 230-3) is in the same

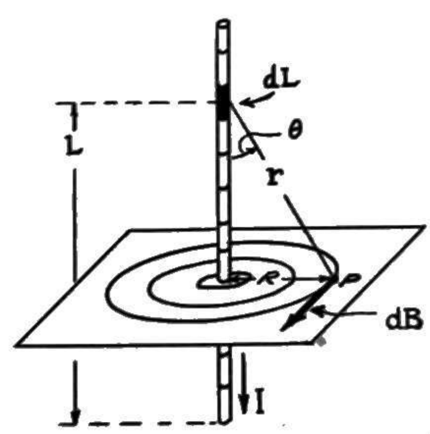

Fig. 230-3

direction as the field due to a long straight wire (Fig. 230-2) of which the segment would be a part. The numerical value of μ_0 will depend upon the units in which it is expressed, and these in turn will be determined by the units in which the other quantities are given in Eq. (230-1).

The value of the proportionality factor μ_0 as it is defined by Eq. (230-1) determines how much flux density B will be established in empty space by a given current. It is therefore referred to as the "magnetic" permeability of empty space. Its numerical value for empty space is 1.26×10^{-6} in mks units, and the value for air is the same through three digits. The numerical factor 4π has been arbitrarily included along with μ_0 in Eq. (230-1) for reasons that appear in more advanced equations. For convenience here, the factor $\mu_0/4\pi$ will be represented by a single symbol b in some of the following discussions.

231. The Magnetic Field of a Flat Circular Arc. If the wire of Fig. 230-1 is bent into a horizontal circular loop about a center P as shown in Fig. 231-1 all parts of the circular loop of wire will cooperate equally to produce a strong resultant downward field at the center point P.

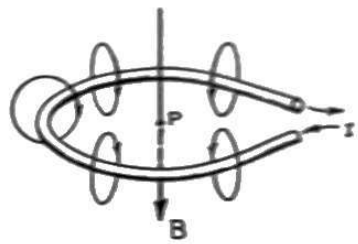

Fig. 231-1

The resultant field at the center can be computed by considering the arc to be made up of segments as indicated in Fig. 230-2. The field dB_1 due to any one segment dL_1 will be given by Eq. (230-1). The distance r_1 is perpendicular to dL_1, and hence $\sin\theta$ is unity. Thus

$$dB_1 = b\,\frac{IdL_1}{r_1^2} \qquad (231\text{-}1)$$

All the segments will contribute fields at P which are in the same direction, and hence the total field B may be found by simply adding the fields from the individual segments. Thus we may write

$$B = b\,\frac{IdL_1}{r_1^2} + b\,\frac{IdL_2}{r_2^2} + b\,\frac{IdL_3}{r_3^2} + \dots \qquad (231\text{-}2)$$

with enough added terms to include one for each segment. Now b, I, and r the same for all segments, and we may factor them out to give

$$B = b\,\frac{I}{r^2}\,(dL_1 + dL_2 + dL_3 + \dots) \qquad (231\text{-}3)$$

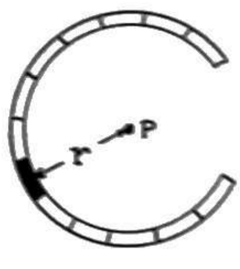

Fig. 231-2

The sum of the lengths of all the separate segments is the total length L of the wire, and hence

$$B = b\,\frac{IL}{r^2} \qquad (231\text{-}4)$$

Equation (231-4) above may be written in a slightly different form by substituting $2\pi Nr$ for L, where N is the number of turns in the coil. This substitution gives

$$B = b\,\frac{2\pi NI}{r} \qquad (231\text{-}5)$$

Both Eqs. (231-4) and (231-5) apply to a piece of wire coiled around in the arc of a circle even if there is less than one full turn. In that case the N of Eq. (231-5) will be a fraction of a turn.

Note that Eq. (231-5) gives the flux density at the center of the coil only. There is no simple formula for computing the flux density at other points, but the general configuration of the lines of force about a coil can be found by a compass or by iron filings. Fig. 231-3 shows the lines of force in a plane perpendicular to the plane of a circular coil. Note that for any point on the axis of the coil, the magnetic field will be directed along the axis.

232. Magnetic Field of a Current in a Straight Wire. The field due to a current flowing in a straight piece of wire can be computed indirectly by the general equation given in Sect. 230. The total field at any point will be the sum of the contributions of different segments of the wire. The addition of the separate terms from the various segments is somewhat more complicated in this case than it was for a circular arc, because each segment will have a different value of r and a different value of θ. However the summation can be performed by calculus to give a relatively simple expression for the total field. It can be shown[1] that a current I in a straight wire AD as shown in Fig. 232 will produce a field at a point P given by the equation

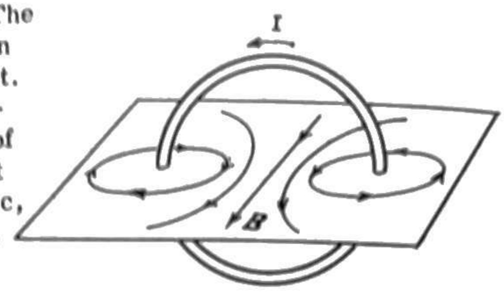

Fig. 231-3

$$B = b\frac{I}{R}(\cos\theta_1 + \cos\theta_2) \qquad (232\text{-}1)$$

Here R is the perpendicular distance from the point P to the wire, and θ_1 and θ_2 are angles as indicated. If the wire extends in both directions for distances large compared to R, the two angles θ_1 and θ_2 will approach zero and the expression for B approaches

$$B = b\frac{2I}{R} = \frac{\mu_0}{4\pi}\frac{2I}{R} \qquad (232\text{-}2)$$

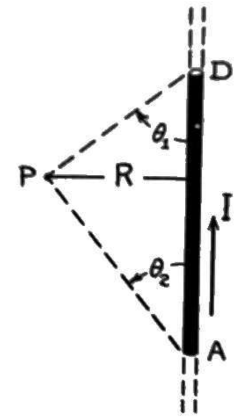

Fig. 232

According to Sect. 230, each segment of the wire AD in Fig. 232 contributes a field at P which is tangent to a circle about AD as an axis. Hence the total field at P will be the simple sum of the separate fields from all the segments. The resultant field at P for the whole wire AD will thus be directed out from the plane of the paper according to the right-hand rule.

Many actual arrangements of wires can be considered as combinations of straight pieces of wire. In that case the total flux density at any point can be computed by a vector addition of the fields for the straight portions taken separately. For example, the field at any point due to a rectangular coil will be the resultant of the four fields from the four straight sides of the coil.

233. The Magnetic Field of a Solenoid. A solenoid is a cylindrical coil of wire made by winding a long piece of wire in a closely wound spiral around an insulating cylinder. The flux density at a point inside the solenoid will be greater than that due to any one turn, because the turns extending in both directions from P will all cooperate to produce an additive effect. As might be expected from this, the more closely the turns are crowded on the spiral, the greater will be the flux density inside the solenoid. It is found that if n is the number of turns per unit length of the solenoid, the flux density B at a point P inside the solenoid be given approximately by the equation

$$B = b\,4\pi nI = \mu_0 nI \qquad (233)$$

This approximation will be close provided the distance from P to either end of the solenoid is large compared to the diameter of the solenoid. For example, if the distance from P to the nearest end is more than five times the diameter, the error in using Eq. 233 will be less than 1%. Also when the point P is relatively far from either end, the flux density will be uniform over the entire cross-section of the solenoid, and parallel to its axis.

For a point on the axis at one end of a long solenoid, the field is one-half of the value given by Eq. (233). This follows because the flux-producing turns extend in only one direction from

1. See Appendix C.

the point in question, instead of extending symmetrically in both directions. Since the field at the end of a solenoid is less than it is at the center, some of the lines of flux which pass through the center must diverge out through the sides of the solenoid before they reach the ends as shown in Fig. 233. The actual configuration of the lines of flux in and around a solenoid may be determined experimentally by a small compass or by using iron filings. The direction of the field in a solenoid may be found by the right hand rule as applied to any one turn.

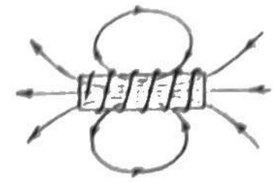

Fig. 233

234. Summary of Equations for Computing Fields Due to Currents. The equations given above for computing the fields due to currents in various geometrical arrangements may be summarized as follows. Here the b is replaced by $\mu_o/4\pi$, as given originally in Eq. (230-1)

General Equation $$dB = \frac{\mu_o}{4\pi}\frac{IdL\sin\theta}{r^2} \qquad (234\text{-}1)$$

At the center of a circular coil $$B = \mu_o \frac{NI}{2R} \qquad (234\text{-}2)$$

Beside a long wire $$B = \frac{\mu_o\, 2I}{4\pi\, R} \qquad (234\text{-}3)$$

Beside a wire of finite length $$B = \frac{\mu_o\, I}{4\pi\, R}(\cos\theta_1 + \cos\theta_2) \qquad (234\text{-}4)$$

Inside a long solenoid having N turns in a length L $$B = \mu_o \frac{NI}{L} \qquad (234\text{-}5)$$

The units will balance in these equations with B in web/m², I in amperes, and all distances in meters, where

$$\mu_o = 1.26 \times 10^{-6} \frac{\text{web per m}^2}{\text{amp per m}} \qquad (234\text{-}6)$$

235. Equivalence of Coils and Magnets. The configuration of the field of a solenoid is similar to that of a bar magnet, as may be seen by comparing Fig. 233 above with Fig. 193-4 of Chap. 19. As might be expected from this, it is found that a current carrying solenoid will exhibit the characteristic properties of a bar magnet. A north "pole" will be observed at the end of the solenoid from which the lines of force diverge, and a south pole at the other end. These poles will exert attractive and repulsive forces on the poles of a magnet according to the usual rule that like poles repel and unlike poles attract. Bits of iron will be attracted to the ends of the solenoid but not to the middle. Suspended horizontally in the earth's field, the solenoid will tend to turn its north pole to the north, like a compass needle.

A flat coil is equivalent to an extremely short solenoid, and it acts like a very short bar magnet having a length which is less than its diameter as shown in Fig. 235. Placed in a magnetic field, such a magnet would tend to turn its north pole face in the direction of the field. So also a flat coil placed in a magnetic field tends to turn so that the face from which the

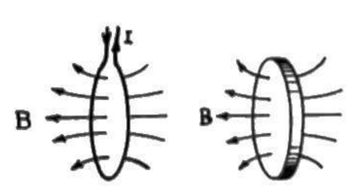

Fig. 235

lines diverge is in the direction of the field. This means that the coil will tend to turn until the flux of coil is in the same direction as the field in which it is placed. (See Sect. 203)

The similarity between the behavior of coils and magnets will be referred to later in discussing the theory of magnetic materials.

236. Resultant Flux of a Current in a Uniform Field. If a straight wire carries a current perpendicular to a uniform field, the lines of flux of the current will be circles superimposed on the straight lines of the uniform field. This situation is illustrated in Fig. 236a, where the uniform field is directed upward, and the wire is shown in cross-section with the current moving toward you. The resultant field is shown in Fig. 236b. On the right side of the wire the two fields add together to give a relatively strong resultant. There the lines of flux are spaced closer together. On the left side of the wire, the two component fields oppose each other, and the resultant field is relatively weak.

Fig. 236a suggests a simple rule for finding the direction of the force of a magnetic field on a current. The force will always be away from the side where the resultant field is strong. This rule is consistent with the rule given in Sect. 200.

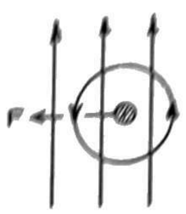

Fig. 236a

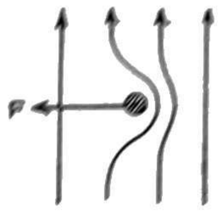

Fig. 236b

Chapter 24

FORCES BETWEEN ADJACENT WIRES

240. Forces between Parallel Wires. Electric currents flowing in adjacent pieces of wire will exert mutual forces on each other. For example, if two currents flow in the same direction through parallel wires, the wires will attract each other. If the currents flow in opposite directions, the wires will repel each other.

An equation for the force between two long straight parallel wires a distance R apart can be derived easily. Let us label the parallel wires A and X respectively as indicated in cross-section in Fig. 240. Assuming that the current is flowing away from the observer in each wire, the lines of flux due to the first wire A will be clockwise circles about the wire as shown. Thus the field due to A will act downward at X to produce a sidewise force on X. According to Sect. 232, this field at X due to A will be given by the equation

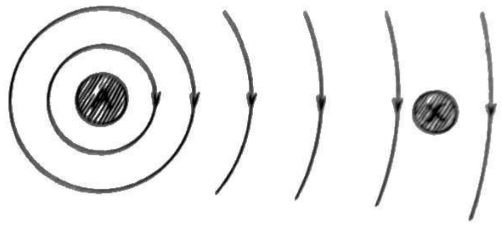

Fig. 240

$$B = b\,\frac{2I_1}{R} \tag{240}$$

where I_1 is the current in the wire A. Then according to Sect. 201, the force of this field on any segment of X of length L will be given by the equation

$$F = BI_2L \tag{241}$$

where I_2 is the current in the wire X. Substituting the value of B from Eq. 240 in Eq. 241 gives

$$F = b\,\frac{2I_1I_2L}{R} \tag{242}$$

The right hand rule (Sect. 200) shows that this sidewise force on X will be directed towards the first wire A.

If the action between the two currents is considered by taking the currents into account in the reverse order, the same expression (Eq. 242) will be obtained. In other words, the two wires will attract each other with equal and opposite forces, in agreement with the law of reaction.

Eq. 242 will also give the force between two parallel wires with currents in opposite directions. For example, if the current in the wire X of Fig. 240 is reversed, the magnitude of the sidewise force on X will be the same, but it will be directed away from A. Thus two parallel currents in opposite directions will repel each other.

241. Watt-meters. The force of one current on another makes possible the construction of a moving coil wattmeter that will give a direct indication of the amount of power expended in a conducting path. The wattmeter is constructed like a moving-coil current meter (Sect. 94) except that the magnetic field in which the moving-coil turns is furnished by a fixed-coil instead of a magnet. The deflecting torque will be proportional to the current I_1 in the moving coil and to the field B produced by the fixed coil. The field B will in turn be proportional to the current I_2 in the fixed coil. Thus the torque will be proportional to the product I_1I_2.

To use a wattmeter for measuring the power expended in a given path, the fixed coil is connected in series with the path. At the same time the moving coil is connected in parallel just as a voltmeter would be connected to measure potential difference. The current I_2 in the moving coil will then be proportional to the difference in potential V across the path, while the current I_1 will be the current through the path. Thus the deflecting torque, which is proportional to I_1I_2 will also be proportional to the power IV in the path. Commercial wattmeters are usually provided with two pairs of terminals, one for each coil. The scale may be marked to read directly in watts by properly spacing the divisions marks at different intervals on different parts of the scale.

Chapter 25

ELECTROMAGNETIC INDUCTION

250. Early in the nineteenth century it was discovered by Faraday in England and by Henry in America that an electromotive force could be induced in a coil by changing the amount of magnetic flux through the coil. For example, if a flat coil is moved in the field of a magnet in such a way that there is an increase or a decrease in the number of lines flux through the plane of the coil, an emf will act around the coil. This emf depends only on the rate of change of the flux, and acts only while the flux is changing. A constant amount of flux through a coil will not produce an emf, no matter how much flux there may be. Emfs produced by the action of a magnetic field are called induced emfs, and the process is called electromagnetic induction.

One of the ways of changing the flux through a coil of wire is to rotate the coil in a magnetic field, and the production of emfs by the rotation of coils in strong magnetic fields is one of the most important of all our industrial processes today. The devices for accomplishing this are known as generators or dynamos. They range in size from the huge turbine driven generators of our power plants down to the small generators which are a part of the electrical systems of present day motor cars.

251. The General Equation for Induced Emfs. As stated in Sect. 250, there will be an emf induced in any circuit when the amount of flux changes in the circuit. There are many ways in which the flux through a circuit can be changed, but there is one general relationship which holds for all. In every case, the induced emf E is found to be equal to the rate of change of the flux through the circuit according to the equation

$$E = -\frac{d\Phi}{dt} \qquad (251\text{-}1)$$

Here $d\Phi$ is the change in flux which occurs in a very short time dt, so that $d\Phi/dt$ is the instantaneous rate of change. If the flux changes from a value Φ_1 to a value Φ_2 in a time t, the average emf will be equal to the average rate of change of flux as given by the equation

$$E = -\frac{\Phi_2 - \Phi_1}{t} \qquad (251\text{-}2)$$

The negative signs in these equations are needed to indicate the direction of the induced emf. One direction around a circuit must arbitrarily be chosen as positive, and any flux is then considered positive if it passes through the circuit so that the right hand screw relationship holds between the positive direction around the circuit and the positive direction through the circuit, as illustrated in Fig. 251.

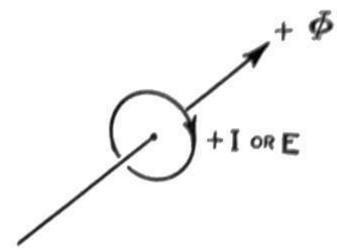

Fig. 251

If the same amount of flux passes through each turn of a coil having N turns, Eq. (251-2) will give the emf induced in each turn. The emfs in the several turns will add together in series, so that the total emf in the coil will be N times the emf in each turn.

Equations (251-1) and (251-2) imply that an emf can be expressed in terms of a unit of flux divided by a unit of time. Consistent with this, we can show that a weber per second is equivalent to a volt as follows: Starting with Eq. (222-4) we see that

$$1\,\frac{\text{newt-m}}{\text{amp}} = 1 \text{ weber} \qquad (251\text{-}3)$$

Hence

$$1\,\frac{\text{weber}}{\text{sec}} = 1\,\frac{\text{newt m}}{\text{amp sec}} \qquad (251\text{-}4)$$

$$= 1\,\frac{\text{joule}}{\text{coul}} = 1 \text{ volt}$$

Since a weber per second is equal to a volt, we can write

$$E = -\frac{\Delta\Phi}{t} \quad \begin{cases} E = \text{number of volts} \\ \Delta\Phi = \text{number of webers} \\ t = \text{number of seconds} \end{cases} \qquad (251\text{-}5)$$

252. Emf Induced by the Relative Motion of a Magnet and a Coil. If a bar magnet is inserted in a coil starting from the position shown in Fig. 252(a) and ending as shown in Fig. 252(b), the final flux Φ_2 through the coil will be much larger than the initial flux Φ_1. If this change occurs in a time t, the average emf acting around in each turn of the coil during this time will be given by the general equation

$$E = -\frac{\Phi_2 - \Phi_1}{t}$$

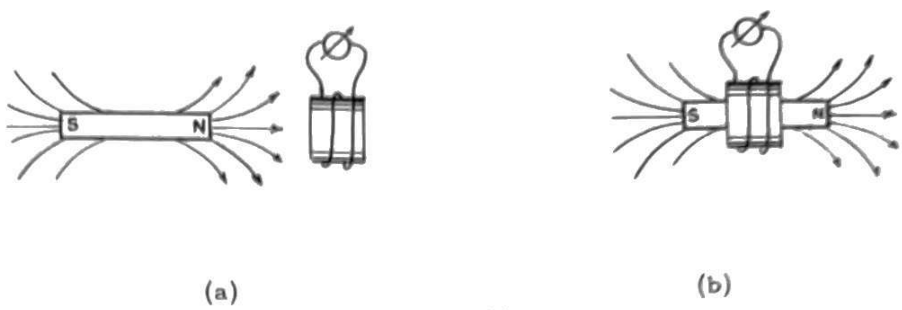

Fig. 252

Such an induced emf may be indicated by a meter connected in series with a coil as shown. The meter will indicate a negative emf while the flux through the coil is being increased. If the magnet is then removed from the coil, the meter will indicate a positive emf while the flux is being removed. If the magnet is allowed to remain at rest in the coil as shown in Fig. 252-b, no induced emf is observed because the amount of flux through the coil remains constant.

The above example of induction also illustrates an alternative way of specifying the direction of an induced emf. This general law states that an induced current is always in a direction to neutralize the change in flux that causes the current. Thus when lines of flux to the right are added by inserting the magnet in Fig. 252(a) the induced current in the coil will produce lines of flux to the left to neutralize those that were inserted. When lines of flux to the right are removed by removing the magnet, the induced emf tends to establish a current to replace the lines that are removed. In other words, the induced current is always in a direction that tends to keep the total flux through the coil constant.

253. Emf Induced by Relative Motion of Two Coils. If a solenoid carrying a current were substituted for the magnet NS in Fig. 252, the situation would not be appreciably different as far as the induction of an emf by relative motion was concerned. This is true because the magnetic field of a solenoid is similar to the magnetic field of a bar magnet, as explained in Sect. 235.

254. Emf Induced by a Changing Current in a Nearby Coil. If two coils are placed close together, it may be that a current in the one coil will produce lines of flux which pass through the other. In that case, the amount of flux through the second coil will change whenever the current in the first coil changes, and an emf will be induced in the second coil. For example, if a solenoid AB with a current I_1 is placed inside a coil CD as shown in Fig. 254, the flux in the solenoid due to I_1 will pass through the coil CD. An emf will be induced in the outer coil whenever the current I_1 in the solenoid increases or decreases but no emf will be induced by a constant amount of flux due to a constant current. According to the rules given above, the emf induced by an increasing current in the solenoid will be in a direction opposite to the current, and that induced by a decreasing current will be in the same direction as the current.

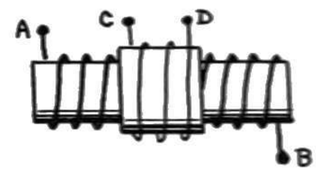

Fig. 254

255. Emf Induced by Rotating a Coil in a Magnetic Field. If a flat coil is rotated in a magnetic field so that the amount of flux through the coil changes, there will be an induced emf in the coil. If the coil is turned from a position perpendicular to the field to a position parallel to the field, the flux will change from some maximum value Φ_0 to zero. If the coil is turned from one perpendicular position to another one where the flux is passing through in the opposite direction, the flux will change from $+\Phi_0$ to $-\Phi_0$. The total change in flux will then be $-2\Phi_0$.

Note that if a flat coil is moved through the uniform earth's field without changing the angle between the plane of the coil and the field, there will be no induced emf because the total number of lines through the coil will be constant. The same lines may not pass through the coil in different positions as the coil is moved in this way, but the total number will be constant if the coil is moved without rotation.

256. Motional Emf. If a conductor such as a piece of wire is moved through a magnetic field, there will be a so-called motional emf induced through the conductor in a direction which is perpendicular to the field and to the direction of the motion. This motional emf acts in one given direction, and hence is somewhat different from the circulatory emf induced in a circuit when the flux changes in the circuit. However many of the same general principles apply to both types of emf, and some of these principles can best be illustrated by motional emf.

A motional emf can be explained in terms of the effect of a magnetic field on moving charges, as discussed in Sect. 206. Let us consider a piece of wire CD which extends north and south, and which is being moved sideways to the east through a vertical field as shown in Fig. 256-1. There are both positive and negative charges in the wire, and these charges will be carried with the wire as it moves. If the wire is moved to the east with a velocity v, a southward force F will act on any positive charge q in the wire according to the equation F = qvB from Sect. 206. This force tends to drive positive charges along the wire from C to D and therefore constitutes an electromotive force along the wire.

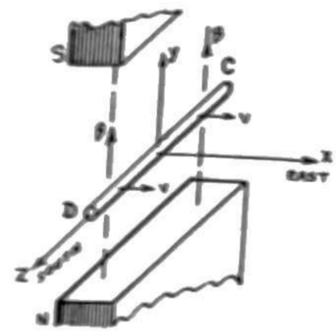

Fig. 256-1

If the piece of wire CD in Fig. 256-1 above has no other wires connected to its ends, the emf induced by the motion will merely crowd the positive charges over to the end D, and keep the end D at a higher potential than C as long as the motion is continued. If a conducting path be provided from D around to C, forming a closed circuit, the charges will flow around in this circuit. Like any emf, this emf will give electrical energy to the charges that pass through it. Since this energy comes from mechanical work done in carrying the wire through the field, such motion may serve to transform mechanical energy into electrical energy.

Fig. 256-2 illustrates a very simple way to provide a conducting path between the ends of a wire CD moving crosswise through a magnetic field. As indicated, the wire CD may be made to slide along on the two sides of a stationary U-shaped piece of metal. An emf E induced from C to D in the moving wire will then produce a current I around in the circuit CDGC. If a current I flows in the direction of the emf as indicated, the energy W expended by the emf in a time t will be

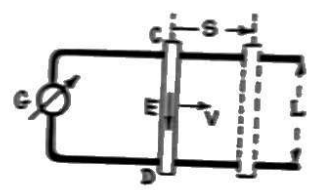

Fig. 256-2

$$W = EIt. \tag{256-1}$$

Because this current flows through the wire from C to D in the magnetic field, there will be an electromagnetic force F = BIL back to the left. In other words, when a current flows due to the emf induced by moving the wire, the field will exert a force on the current to oppose the motion. Thus mechanical work must be done to keep the wire moving with a constant velocity v, even if there is no mechanical friction.

The work W done in moving the wire a distance S will be equal to FS, where F = BIL. Hence we may write

$$W = BILS \tag{256-2}$$

According to the principle of conservation of energy, this mechanical work done must be equal to the electrical energy which is produced in the emf, so that Eqs. 256-1 and 256-2 may be combined to give

$$EIt = BILS \tag{256-3}$$

An expression for the magnitude of the emf may be obtained from Eq. (256-3) by canceling the I and writing

$$E = BLS/t. \tag{256-4}$$

Also since S/t is the velocity with which the wire is moving, we may write

$$E = BLv \tag{256-5}$$

showing that the emf is proportional to the velocity.

Note that the same emf will be induced whether a current flows or not, but the amount of force required to keep the wire moving with a constant velocity depends on the current. If no current flows, no force will be needed to maintain motion, no mechanical work will be done, and no energy will be expended by the emf.

The rotation of certain types of commercial generators amounts to carrying the sides of its coils crossways through a magnetic field. The emf in this case can be considered either as a motional emf, or as a circulating emf induced by the change in flux through the rotating coils. In other words, equations 256-5 and 251-1 both apply equally well.

257. Motional Emf and Magnetic Flux. When a wire moves transversely through a magnetic field with a constant velocity as shown in Fig. 256-1 or 256-2 above, we may consider that it "cuts" the lines of magnetic flux like a swinging scythe cuts stalks of grass. Thus we may say that an emf is induced whenever a wire moves so as to cut lines of flux. Since the magnitude of the emf is proportional to v, it is also then proportional to the number of lines of flux cut per second.

As the wire of Fig. 256-2 moves to include more area in the circuit CDGC, it will also include more flux. If the wire of length L moves a distance S, the area added to the circuit will be LS. Thus Eq. (256-4) may be written in the form

$$E = \frac{B\text{ (added area of circuit)}}{t} \tag{257-1}$$

Now the product of B and the added area will be the added flux included in the circuit. This added flux may be written as the difference between the total final flux Φ_2 and the orginal flux Φ_1, so that

$$-E = \frac{\Phi_2 - \Phi_1}{t} \tag{257-2}$$

This is the same as the general equation for an emf induced in a circuit as given in Sect. 251.

258. Lenz's Law is a general law that applies to all cases where an emf is induced by mechanical motion. It states that the direction of any current induced by mechanical motion is in a direction to give a force that will oppose the motion. This law applies when a motional emf is induced in a moving conductor (Sect. 256). It also applies when an emf is induced by relative motion between a coil and a magnet or between two coils (Sects. 252 and 253). For cases where it applies it agrees with other rules previously given for the direction of the induced emf.

Lenz's law may be deduced from the principle of the conservation of energy. If an induced emf drives a current, it expends energy. This energy comes from mechanical motion, and hence there must be an opposing force against which work can be done when the motion occurs.

259. Eddy Currents. Circulating currents can be induced in the material of a conducting body by changes in flux. For example, suppose a solid metal disk D is rotating clockwise about a horizontal axis A, with a fixed pole of a magnet placed so that its lines of force pass horizontally through the lower part of the disk. The region where the lines of magnatic flux B come through the disk is indicated by dots on Fig. 259. Since the disk is made of one solid piece of metal, any closed path such as P or Q marked on the metal will constitute a closed conducting path around which charges may flow. Note that as the circuit Q moves, the number of lines of flux coming out through it will increase, and hence there will be a circular flow of charge around in the metal of the disk as indicated by the arrow tips. Similarly, the number of lines coming out through P will decrease as it moves, and the induced current will flow around in this circuit

in the opposite direction. It follows that as the disk rotates, there will always be charges flowing up through the magnetic field B from the edge of the disk to the center. Since a current is flowing up through a magnetic field which is directed out from the drawing, the field will exert a force on the charges to the right, thereby tending to keep the disk from rotating.

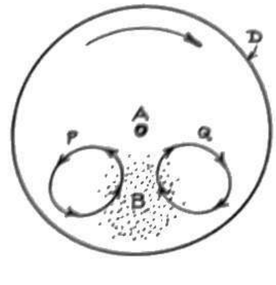

Fig. 259

Currents which circulate around in a solid mass of metal like those described in the above example are called eddy currents. In general, eddy currents will be induced in a solid piece of metal whenever there is relative motion between the metal and a non-uniform magnetic field. Note that if the field in the above illustration had been equally strong everywhere, the total number of lines through a circuit such as Q would be the same for all positions of Q, and no emf would be induced as the disk moved. Whenever eddy currents are induced by relative motion,the resulting current flow will be such as to tend to prevent the relative motion, according to Lenz's law.

Eddy currents may also be induced in a solid piece of metal without motion if the metal is placed in a magnetic field which increases or decreases in magnitude.

Chapter 26

MEASUREMENT OF FLUX BY INDUCTION

260. Flow of Charge with an Induced Emf. As explained in Sect. 251, the average emf induced in a coil of N turns by a change in flux from Φ_1 to Φ_2 is given (without regard to sign) by

$$E = N(\Phi_2 - \Phi_1)/t \tag{1}$$

where t is the time required for the change in flux to occur. If the ends of the coil are connected through a conducting path to form a closed circuit having a total resistance R, the average current flowing will be given by

$$I = E/R = N(\Phi_2 - \Phi_1)/Rt. \tag{2}$$

Multiplying both sides of the equation by t gives

$$It = N(\Phi_2 - \Phi_1)/R. \tag{3}$$

Since the average current I multiplied by the time gives the total charge Q which has passed any point in the circuit, we may write

$$Q = N(\Phi_2 - \Phi_1)/R \tag{4}$$

This equation shows that the charge is proportional to the change in flux, and independent of the amount of time required to make the change.

261. Ballistic Galvanometer. If a steady current is started through a moving coil galvanometer, there will be a small lapse of time before the coil will reach the deflection which corresponds to that current. The delay in this response is due largely to the inertia of the coil. For the same reason a deflected meter will return some what slowly to its zero position after the current has been stopped. If a momentary current is sent through a galvanometer, the deflection may never catch up with the current while it lasts, and may persist for a while after the current

has stopped. Thus for measuring currents of short duration, a moving-coil meter is ineffective.

Although a galvanometer may be useless as a current-measuring device when the current lasts only for a short time, there is one condition under which it may be useful in an entirely different way. If the flow of current lasts such a short time that it has stopped completely before the galvanometer coil has time to move appreciably, then the subsequent motion of the coil will indicate the total amount of charge which passed through the galvanometer during the temporary flow of current. As the surge of current passes through the coil, it gives the coil an impulse like the sharp blow of a hammer. The coil subsequently moves under this impulse and swings around until it is stopped and returned by the restoring torque of the suspension. The maximum deflection reached by the freely swinging coil is then proportional to the total charge that passed through the galvanometer. Thus a galvanometer can be used to measure charge if the charge passes through quickly enough. It is then said to be used as a ballistic galvanometer. The name comes from a similarity to the ballistic pendulum used to measure the velocity of bullets.

Sensitive, suspended coil galvanometers are most effective as ballistic galvanometers. Although a given galvanometer may be used either as a current galvanometer or as a ballistic galvanometer, the coil of an instrument intended primarily for ballistic use generally has a relatively large inertia. These meters will operate satisfactorily to measure a charge that does not require more than about a half a second to pass through the meter.

The ballistic sensitivity of a galvanometer is defined as the amount of charge per unit scale division. Galvanometers with a sensitivity as little as .002 microcoulombs per division are commercially available. A ballistic galvanometer can be calibrated by sending known charges through the galvanometer and noting the corresponding deflections, or it can be computed from the current sensitivity if the mechanical characteristics of the coil are known.

262. Flux-meters. The flux through a coil can be measured by connecting the coil in series with a ballistic galvanometer, and then removing the flux quickly from the coil. If the flux is removed quickly enough so that the galvanometer responds as a ballistic galvanometer, the resulting maximum deflection will be proportional to the change of flux in the coil. This is true because the deflection of a ballistic galvanometer is proportional to the charge which passes through the galvanometer, and because any charge circulated by an induced emf is proportional to the change in flux in the coil according to the equation

$$Q = N\frac{(\phi_2 - \phi_1)}{R}$$

Since the induced current must flow through the resistance of the coil and galvanometer in series, the R of this equation must include the resistance of the galvanometer along with that of the coil.

A so-called search coil and a ballistic galvanometer used together in this way may be referred to as a flux-meter. The number of lines of flux corresponding to each division of the meter scale is called the calibration constant of the flux-meter. This constant can be determined experimentally by observing the deflection produced by a known change in flux.

Chapter 27

THE MAGNETISM OF MATERIAL BODIES

270. The magnetism exhibited by a permanent magnet is a property that can be gained or lost by the material. For example a piece of ordinary iron can be made a magnet by simply putting it in a magnetic field. Also a so-called permanent magnet can be demaganetized, leaving a piece of the same material without magnetism. In the following sections we will discuss the nature of magnetism, and show how the magnetism of material bodies can be explained in terms of electric currents inside the material.

271. Induced Magnetism. As stated above, a piece of iron can be magnetized by placing it in a magnetic field. The magnetizing field used may be a field produced by a current, or it may be a field due to another magnet. The newly created magnet will have its south pole where the lines of flux of the magnetizing field enter the piece of material, and its north pole where they leave. If the magnetizing field is due to another magnet, this is equivalent to saying that the newly formed poles are opposite in sign to the nearest pole of the original magnet. For example, Fig. 271 shows how a permanent horseshoe magnet would create the poles N' and S' on a bar of unmagnetized material. The poles NS of the original magnet will then attract the adjacent opposite poles N'S' of the newly created magnet. Thus it is that a permanent magnet can attract a neutral piece of iron by creating poles on it.

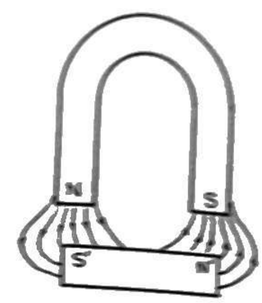

Fig. 271

A magnet created by placing a piece of material in a field is said to be "induced" by the field, and the process is called magnetic induction.

Magnetic materials may be magnetized in the earth's field, but this field is so weak that its effect is not always noticeable. In places like shipyards however where large pieces of steel lie in a given position while they are subjected to vibration and heating, a noticeable magnetization may result.

272. Permanent Magnetism. Materials which retain some of their magnetism when removed from a magnetizing field are said to be permanently magnetized. This magnetism may be retained indefinitely if the material is carefully handled. It may be lost if the material is jarred, heated or subjected to a sufficiently strong reversed field.

Common commercial magnets are made from materials which will retain a large amount of permanent magnetism, and they are magnetized by placing them in strong magnetic fields. Although pure iron can be strongly magnetized, it will not retain much of that magnetization permanently and hence it is not used for permanent magnets. Alloys containing cobalt and nickel have been developed which retain a large amount of magnetism persistently, and such alloys have been widely used in making permanent magnets.

The magnetism of a permanent magnet may be decreased or completely reversed if it is placed in a strong field which tends to magnetize it in the opposite direction. For example, if the north pole of a strong magnet is brought up to the north pole of a compass needle without allowing time for the compass needle to turn, the strong magnet may reverse the magnetism of the compass needle. A south pole will be induced on the end of the compass needle where the north pole was before.

273. Electromagnets consist essentially of a coil of wire wrapped around a core of some magnetic material such as a piece of iron. When a current flows in the coil, the core becomes magnetized so that it serves as a magnet producing a much stronger field than the coil would produce by itself. Electromagnets are in general stronger than permanent magnets of the same size because magnetic materials do not permanently retain all the magnetism which they can be

made to possess while they are in the magnetizing field. If the core of an electromagnet is made of a material which loses nearly all its magnetism when the magnetizing field is turned off, the electromagnet can be turned "on" and "off" by remote control, thereby giving it an advantage over a permanent magnet for some uses.

274. The Amperian Theory of Magnetism. As pointed out in Sect. 235, the field of a magnet resembles the field of a solenoid. It also appears that the origin of the field may be fundamentally the same in both cases. In the solenoid, the field is due to electrical currents flowing around in the turns of the solenoid, and the observed field is the resultant field due to all of the successive turns of the solenoid. In a magnet, it appears that the field is also due to the motion of electricity in circular paths. This motion has been identified with rotary motion of spinning electrons. These spinning electrons are assumed to rotate incessantly inside the atom just as the earth rotates about its axis day after day. In an unmagnetized piece of material, the axes of spin for the different electrons are oriented more or less at random, so that there is no combined effect. In a permanent magnet, there are enough of the electrons spinning in a given direction so that there is a marked external effect. Years before there was any knowledge of spinning electrons, it was deduced by Ampere that there must be some sort of elementary circuits in the interior of magnetic materials which corresponded to the separate turns of a solenoid. For that reason, these elementary circuits which are now identified as spinning electrons are still referred to as Amperian currents.

This theory of magnetism is consistent with the fact that magnetic materials may be magnetized by placing them in a magnetic field. The magnetizing field exerts a torque on the spinning electrons just as it would on a coil of wire, and tends to turn them so their fields will be in the same direction as the magnetizing field. If a large number of spinning electrons all have their axes of spin lined up in the same way, they will cooperate to produce a strong magnetic field. If the nature of the material is such that this arrangement of the spinning electrons persists after the magnetizing field is removed, the piece of material will then be permanently magnetized.

The effect of heat on a magnetic material is also consistent with the idea that magnetization involves some sort of molecular or atomic rearrangement. The molecular energy associated with high temperatures tends to break up any existing arrangement of the amperian circuits. Thus the magnetism of a permanent magnet may be destroyed by heating. If a magnetic material is to be magnetized in a magnetizing field, a temporary application of heat may aid the magnetizing field by breaking up previously existing arrangements. Thus the strongest permanent magnets are made by allowing them to cool from a high temperature while they are in a magnetizing field. The effect of jarring and vibration on a magnetic material is similar to the effect of heat, as might be expected since higher temperatures are associated with higher degrees of molecular vibration.

275. The Nature of Magnetic Poles. Magnetic poles have been referred to as regions on a magnet where the magnetic forces appear to be concentrated. Two kinds of poles have been recognized, depending on the way the magnet points in a magnetic field. On plots that show the field of a magnet, a north pole appears as a region from which lines of flux diverge, and a south pole appears as a region to which lines converge. For many years it was assumed that magnetic fields originated from elementary poles, just as electric fields originate from electrons and protons. In the light of Amperes theory of magnetism as verified by many experiments, it appears now that magnetism may be considered as a subdivision of the general phenomena of electricity, rather than as a separate phenomena.

It is certainly true that magnetic poles as observed on magnets can exist without any elementary poles inside the magnet. A current carrying solenoid exhibits north and south poles as far as external effects are concerned, and there we know that the lines of force which converge in to the south pole end do not end there. Rather they pass unbroken through the solenoid and diverge from the north pole end as illustrated in Fig. 233. Hence magnetic poles as observed from the exterior of a bar magnet do not necessarily require the interior existence of elementary

poles on which the lines of magnetic force can end. Instead we can consider a magnet as a kind of solenoid where the aligned Amperian circuits constitute the turns of the solenoid.

In discussing the nature of magnetic poles, it is of interest to note that an isolated pole of either sign cannot be obtained by breaking a magnet in two. If a magnet is broken in an attempt to separate the two poles, new poles appear on the broken parts so that each part has both a north and south pole after they are separated. This behavior is illustrated in Fig. 275-1. Another point of interest is that it has never been found possible to make a magnet with one pole stronger than the other, although one pole may be spread over more area than the other. Magnets made with one pole of each kind concentrated at the two respective ends are most common because they are most useful. Magnets can be made with either pole spread over considerable area, or one pole can be distributed over separate areas. Thus a bar magnet can be magnetized with a north pole at each end and a south pole in the middle as indicated in Fig. 275-2. In any case however, the total number of lines of flux coming out of all the north pole areas on a magnet is always equal to the total number of lines entering the south pole areas. These experimental observations support the idea that the lines of force which enter the south pole of a magnet must continue unbroken through the magnet and emerge some where else, forming endless loops as they should according to the Amperian theory.

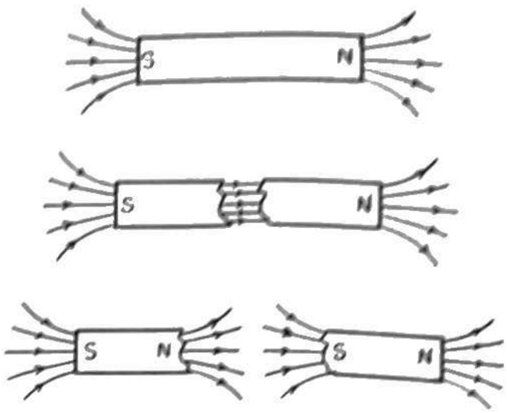

Fig. 275-1

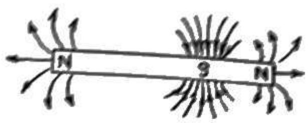

Fig. 275-2

In the same way, it appears that all of the magnetic fields with which we are now familiar involve endless loops of flux produced by moving charges. There is of course the possibility that undiscovered situations may exist in nature where magnetic lines originate or end on elementary poles. Whether or not there are any elementary magnetic poles, the idea of a magnetic pole is still a useful geometrical concept. It can be used as an abstract center for magnetic forces just as the center of gravity is used in mechanics as an abstract center for gravitational forces.

276. Magnetization without Magnetic Poles. It follows from the preceding sections that if the amperian circuits in a body of material are aligned, there will be loops of magnetic flux associated with the material just as loops of flux are associated with solenoids. The existence of these loops of flux is an essential feature of magnetism. On the other hand, the existence of magnetic poles is not an essential feature of magnetism. For example, an endless ring of magnetic material can be magnetized to have loops of magnetic flux but no poles. The magnetism in such a ring is similar to that in a horseshoe magnet with its poles bent close together as shown in Fig. 276. To secure an endless ring without poles, the gap between the poles in Fig. 276 could be closed by bending the ends together. The poles would then neutralize each other, but the magnetism in the ring would continue to exist as loops of flux. Endless magnets without poles will not produce any external field. Although they can not be used to exert attractive forces, they are widely used as cores of transformers (See Sect. 380).

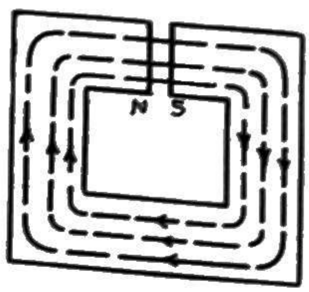

Fig. 276

Chapter 28

THE MAGNETIC PROPERTIES OF MATERIALS

280. Magnetic materials differ with regard to the ease with which they can be magnetized. They also differ with regard to the maximum amount of magnetization that can be produced in a given volume of material, and with regard to the amount of the magnetization that will be retained after the material is removed from the magnetizing field. Only a relatively few materials can be magnetized strongly enough to be of practical interest. These include the elements iron, nickel and cobalt, and a number of different alloys. Because these materials behave in general like iron, they are said to be ferromagnetic. In this chapter we will consider the measurement of the magnetic properties of ferromagnetic materials, and we will also discuss briefly the magnetic characteristics of materials which are not ferromagnetic.

281. The Measurement of Magnetic Properties. To measure the magnetic properties of a material, we need a known magnetizing field in which a sample of the material can be placed, and a means for measuring the resulting flux density in the sample. The calculations required are relatively simple if the magnetizing field is furnished by a long solenoid, and if the sample is in the form of a long bar that completely fills the solenoid. The magnetic flux produced in the bar can then be measured by a ballistic galvanometer with a search-coil placed around the solenoid as shown in Fig. 281-1.

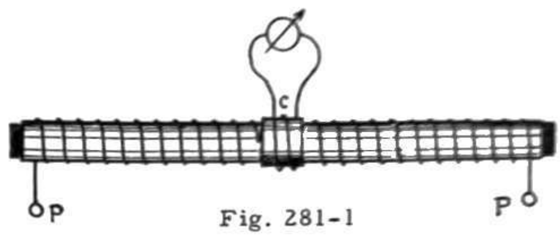

Fig. 281-1

To make simple calculations possible, the solenoid and bar should extend infinitely far in either direction. In practice this condition can be approximated by using a long thin solenoid. The effect of the ends may also be eliminated by making the solenoid and sample in the form of an endless ring, or torroid, as shown in Fig. 281-2. A third possible experimental arrangement is shown in Fig. 281-3. Here a heavy yoke Y of soft iron is bridged across from one end of the solenoid to the other. With this arrangement, the poles of the solenoid are neutralized by the opposite poles which they induce at the ends of the yoke. This makes the field in the solenoid like the field in an endless solenoid.

Fig. 281-2

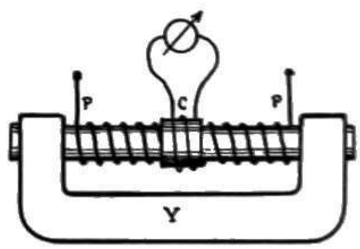

Fig. 281-3

In any of the above experimental arrangements, the search-coil will measure the total flux density B through the coil. This will include the flux density B_0 due to the magnetizing current I in the solenoid superimposed on the flux density B_m due to the amperian circuits in the sample of magnetic material. The total flux density B will then be given by the equation

$$B = B_0 + B_m. \tag{281}$$

The field furnished by the current in the solenoid is the field that tends to allign the amperian circuits in the core, and it may therefore be referred to as the magnetizing field. It is generally desirable to get a large total flux density under the control of the magnetizing field. This is possible with some magnetic materials such as iron, where the contribution B_m of the core material may be thousands of times greater than the direct contribution B_o of the controlling field. The ratio of the total flux density B to the flux density of the magnetizing field is a measure of how much the flux has been increased by the presence of the core, and this ratio is referred to as the relative permeability of the core with respect to empty space.

To illustrate, let us consider a sample problem involving a solenoid 1 meter long with 1200 turns and a cross-sectional area of 2 cm². Consider that the core is originally unmagnetized and that it fills the solenoid so that its cross-sectional area may be taken to be the same is that of the solenoid. Given that the flux meter indicates an increase of 180×10^{-6} webers when a current of 5 amperes is turned on in the solenoid, we can compute the resultant flux density B in the core as

$$B = \frac{180 \times 10^{-6}\ \text{web}}{2 \times 10^{-4}\ \text{m}^2} = .9\ \text{web/m}^2$$

The flux density B_o due to the solenoid alone is given by Eq. (234-5) to be

$$B_o = \mu_o \frac{NI}{L} = (1.26 \times 10^{-6}) \frac{1200 \times 5}{1} \frac{\text{web}}{\text{m}^2}$$

$$= .00754 \frac{\text{web}}{\text{m}^2}$$

The relative permeability is therefore the ratio of .9 to .0075, which is approximately 120 to 1.

282. The Permeability of Magnetic Materials. The flux density in a long uniform solenoid due to the current I is given by Eq. (234-5). If we use B_o to represent this flux density as was done in the preceding section, we can write

$$B_o = \mu_o \frac{NI}{L} \qquad B_o = \mu_o \frac{NI}{L} \qquad (282\text{-}1)$$

If a long uniform core is placed in the solenoid, the total flux density B will in general be larger than B_o, and we can write

$$B = \mu \frac{NI}{L} \qquad (282\text{-}2)$$

where the factor $\boxed{\mu}$ depends on the core material. Just as μ_o is a measure of the permeability of empty space for lines of flux,(1) so μ is referred to as the magnetic permeability of the core material. The ratio of μ to μ_o is the same as the ratio of B to B_o, and it is the relative permeability of the core material as defined above.

283. Magnetization Curves. Equation (282-2) above applies to the behavior of a magnetic core placed in a coil under the control of the magnetizing current in the coil. It is therefore a very important equation from both a theoretical and a practical viewpoint in that it describes the result obtained with any particular core material and for any given value of the magnetization current. The meaning of the equation may be clarified by stating it in words as follows

(1) See last paragraph of Sect. 230.

B			$\frac{NI}{L}$	
The total flux density	=	The permeability of the core material	x	The magnetizing influence of the current I as modified by the number of turns and the length of the solenoid	(283-1)

It is convenient to refer to the third term as a single quantity, and we can write

$$\frac{NI}{L} = H \qquad (283\text{-}2)$$

where H is called the magnetic intensity due to the current I.* As written, the equations (282-2) and (283-2) apply only to a long uniform core in a long uniform solenoid, or to some equivalent arrangement, but corresponding equations can be written for any case where a core is placed under the influence of a magnetizing current. Since B, N, I and L in Eq. (282-2) can all be measured, the permeability μ of a core can be computed by that equation.

The dimensions of a unit for H as given in Eq. (283-2) may be found by applying the equation to compute the value of H in a typical case. Thus if I = 2 amp in a solenoid that has 1500 turns in one meter length L, then Eq. (283-2) gives

$$H = \frac{1500 \text{ turns} \times 2 \text{ amp}}{1 \text{ m}}$$

$$= 3000 \frac{\text{amp-turns}}{\text{m}} \qquad (283\text{-}3)$$

Thus an amp-turn per meter is the mks unit for H. The number of turns N is a purely numerical multiplying factor, and although the work "turn" is often included in the name of the unit, it may be discarded at will. Thus an "ampere-turn per meter" is the same as the "ampere per meter". The "oersted" is a unit of magnetic intensity from the cgs system which is frequently used. Units may be converted by using the relationship

1 oersted = 79.6 amp-turns per meter

It follows that if the magnetic field of the current has a flux density B and a magnetic intensity H, then the value of B in gauss in numerically equal to H in oersteds.

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* A general definition of the intensity H of a magnetic field is beyond the scope of this book. For points in empty space, the intensity H of a given magnetic field is related to the flux density B of the same field by the equation

$$B = \mu_o H \qquad (283\text{-}4)$$

This means that, in empty space, H is a vector quantity having the same direction as B, and a magnitude that is proportional to the magnitude of B. The historical development of our knowledge of magnetism is such that both B and H are used in connection with magnetic fields in empty space, although such a field could be adequately described by either one alone.

The mks unit of H can be found by using the value of μ_o (1.257×10^{-6} weber per amp m) in Eq. (283-4). Thus for a point in empty space where B = 1 weber/m^2, we get

$$H = \frac{B}{\mu_o} = \frac{1}{1.257 \times 10^{-6} \text{ web/amp m)}} = .796 \times 10^{-6} \text{ amp/m}$$

This is consistent with Eq. (283-3), where the same unit appears for H.

The permeability μ of many magnetic materials is different for different values of the magnetizing field, and for this reason, the magnetic nature of many materials cannot be described by giving any one value of μ. Instead, the magnetic nature of a material is best described by a so-called "magnetization curve" of B vs H. The shape of the magnetization curve as shown in Fig. 286 is typical of ferromagnetic materials in general. The curve rises slowly at first because the amperian circuits are locked together by mutual forces in local combinations which produce no external field. After H becomes large enough to break up these local combinations, the circuits will swing around into alignment with the field. The contribution B_m of these circuits to the total induction B therefore increases rapidly as H is increased further, and the curve of B vs H rises sharply. Finally, after the amperian circuits are almost completely aligned, there can be little further increase in their contribution to the total induction. Hence the only appreciable increase in B will come from the flux density B_o which is due to the magnetizing current. This leveling off on the curve after its steep rise is referred to as magnetic "saturation."

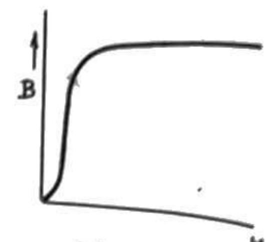


Fig. 286

Very careful observations show that the curve does not rise uniformly at its steepest part. Rather the curve rises in sharp steps on an extremely fine scale. This is as would be expected on the assumption that whole groups, or blocks, of amperian circuits turn around suddenly together, once an original combination is broken up.

For any value of the magnetizing field H represented by a point on a curve of B vs H, the permeability will be given by the slope of a line drawn from the origin to that point. Thus μ may have different values for different points along the curve, and the maximum value will occur at the knee of the curve where it begins to flatten out. Note that this is not the point where the curve is steepest.

284. Magnetic Hysteresis. If a piece of material acquires a flux density (or induction) B according to the curve OA of Fig. 287 when a magnetizing field H is applied, the material will tend to retain some of this magnetization permanently. Consequently if the magnetizing field is reduced, the induction B for any value of H will be larger than it was for the same magnetizing field as the material was being magnetized. Also there will be some magnetization left after H has been reduced to zero, as indicated by the curve AR of Fig. 287. The value of B at the point R where H has been reduced to zero represents the residual flux density B_r retained as permanent magnetization. To reduce this magnetization to zero and to bring the curve down to the point C, a negative magnetizing field must be applied as indicated. This reversed field required to demagnetize a previously saturated piece of magnetic material is called the "coercive force, H_c." The general tendency for changes in induction B to lag behind changes in the magnetizing field H is referred to as hysteresis.

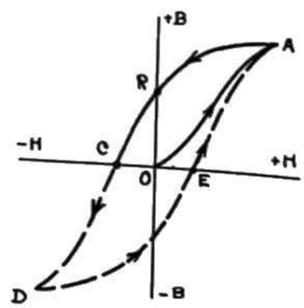


Fig. 287

When magnetic materials are used in connection with alternating currents, the material is magnetized alternately in opposite directions. There the occurrence of hysteresis is objectionable since it causes a waste of energy. The complete loop ARCDEA is a "hysteresis loop" such as results when the field H is decreased from a maximum positive value to an equal negative value, and then restored to its original positive value. The area of the loop represents an amount of energy per unit volume which is lost in heat in the material each time it is carried through such a cycle.

288. Properties of Typical Ferromagnetic Materials. The value of the coercive force for a substance is a good indication of how hard it is to change its magnetic condition, either to magnetize it or to demagnetize it. Usually the material will be approximately saturated by applying a magnetizing field equal to three or four times its coercive force. Values of coercive forces also give a comparative idea as to permeabilities since substances that are hard to magnetize generally have low permeabilities, and vice versa. Some approximate magnetic data are given in Table 288 to show the difference between various ferromagnetic materials. These values are to be taken as approximate values which indicate the order of magnitude only.

Table 288

Approximate Magnetic Magnitudes for Some Materials

	H to produce saturation* $\frac{NI}{L}$	Saturated B*	Coercive force H_c	Residual flux density B_r
	$\frac{\text{amp}}{\text{m}}$	$\frac{\text{web}}{\text{m}^2}$	$\frac{\text{amp}}{\text{m}}$	$\frac{\text{web}}{\text{m}^2}$
Iron	240	2.15	60	.7
Nickel	1200	.60	270	---
Permalloy	16	1.00	3	---
Carbon Steel	---	---	4,800	.90
Alnico	---	---	44,000	1.25

Materials like iron or permalloy with a low coercive force and a high permeability are suitable for cores in electromagnets and other applications where the magnetization is to be controlled by controlling the magnetizing current. Carbon steel and alnico are on the other hand suitable for permanent magnets, because of their relative high coercive force.

289. Paramagnetism and Diamagnetism. As stated before, only a few ferromagnetic materials like iron can be magnetized strongly enough to be of practical importance. Other materials are generally referred to as non-magnetic. However, all materials show some magnetism if they are examined under conditions that permit extremely small effects to be observed. For example, a piece of aluminum will be slightly attracted by either pole of a magnet, although it cannot be magnetized strongly enough to be lifted by a magnet.

The attraction of a magnet for aluminum can be observed by suspending an elongated piece of aluminum between the poles of a strong magnet. The piece of aluminum will then turn parallel to the field just as a piece of iron would do. Although aluminum will be attracted by a magnet in the same way that iron is attracted, the amount of magnetism induced in the aluminum is very small by comparison. For magnetizing fields ordinarily produced by magnets, the induced flux density in a piece of aluminum is only about one billionth as large as it would be in a piece of iron. Materials that are weakly attracted by a magnet like aluminum are said to be paramagnetic.

The magnetism of paramagnetic materials is strictly proportional to the magnetizing field. This means that they do not exhibit any saturation in ordinary fields, and they do not show hysteresis or permanent magnetization.

Paramagnetism is like ferromagnetism, in that it appears to depend on the alignment of spinning electrons in a magnetizing field. In paramagnetic materials, however, the axes of the spinning electrons are more rigidly fixed in direction. Only small changes are produced by the

* These values are for approximate saturation as represented by the knee of the magnetization curve. The curves for magnetically hard materials such as carbon, steel, and alnico have no sharply defined knee.

strongest available fields, and these changes disappear as soon as the field is removed.

Some materials are magnetized differently from iron or aluminum in that a piece of the material will be repelled by either pole of a magnet. Silver, bismuth and glass are typical examples of materials that will be repelled by either pole of a magnet. If a bar of such a material is suspended between the poles of a magnet, it will turn itself cross wise to the field. Materials which are repelled by either pole of a magnet are said to be diamagnetic.

Diamagnetism appears to result from the orbital motion of electrons in atoms. If an electron moving in a fixed orbit is placed in a magnetic field, there will be an induced emf around the orbit according to the ordinary rule for induced emf in a circuit. If the orbit is one that produces a field in the direction of the magnetizing field, the induced emf will decrease the equivalent current in the orbit. If the orbit is one that produces a field opposite to the magnetizing field, the induced emf will increase the equivalent current. Both effects will add together to give a resultant added field opposite to the magnetizing field. Assuming that a random distribution of the orbital motions of all electrons previously gave no field, the added field will appear externally as a field due to the material. In other words, the material will be magnetized, and magnetized in a direction opposite to the magnetizing field. This means that the poles of a permanent magnet will induce similar poles on adjacent parts of a piece of diamagnetic material, and the material will then be repelled.

In general, atomic electrons have both the orbital motion that gives diamagnetism and the spinning motion that gives paramagnetism. Thus all materials are both diamagnetic and paramagnetic. The external effect observed for a given material depends on which effect is larger in that material.

Chapter 29

THE MAGNETIC CIRCUIT

290. In practice we may wish to compute the number of closed loops of flux Φ that will be found in a core as shown in Fig. 290. Here the core is almost a closed ring of material with a narrow air gap, and the magnetizing coil is concentrated at one place.

If the permeability of the core is large compared to the space around it, the loops of flux produced by a current in the coil will all lie in the core as indicated by the dotted line. The core thus serves as a path along which the lines of flux lie. The lines of flux form closed rings, and hence the core and air-gap together may be referred to as a magnetic circuit. In such a magnetic circuit, the amount of flux is related to certain other quantities by an equation that is analogous to ohm's law for an electric circuit. In this analogy, the flux Φ in the magnetic circuit corresponds to the current in the electric circuit. The reciprocal of magnetic permeability corresponds to resistivity. The product of the total number of turns N and the magnetizing current I in the magnetic circuit corresponds to the electromotive force in the electric circuit. The product NI is accordingly referred to as the magnetomotive force.

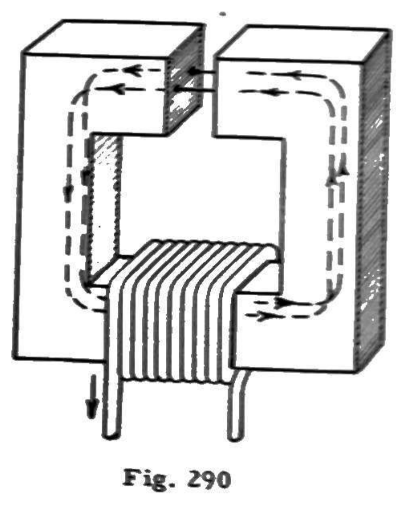


Fig. 290

To illustrate, let us consider the magnetic circuit shown in Fig. 290. The core constitutes a magnetic "conductor" having a cross-sectional area A and a length L_1 bent into circular form.

The section of the circuit through the air gap can then be considered as a short conductor of length L_0, having the same cross-sectional area as the core. Let μ_1 be the permeability of the core material and μ_0 be the permeability of the air in the gap between the ends of the core. If this magnetic circuit was an electrical circuit in which the resistivity of the core material was ρ_1 and that of the air in the gap was ρ_0, we would write

$$I = \frac{E}{\rho_1 \frac{L_1}{A} + \rho_0 \frac{L_0}{A}} \tag{290-1}$$

By analogy we can therefore write

$$\phi = \frac{NI}{\frac{L_1}{\mu_1 A} + \frac{L_0}{\mu_0 A}} \tag{290-2}$$

The denominator, which is analogous to the resistance of the electrical circuit, is referred to as the reluctance of the magnetic circuit. Cores forming a closed ring are widely used and Eq. (290-2) may be applied to a core without any air gap by simply taking L_0 to be zero.

Eq. (290-2) is of considerable practical interest, but it must be remembered that it is approximate. It is derived from a corresponding exact relationship that may be found in more advanced textbooks. For the conditions under which Eq. (290-2) holds, note that the distribution of the N turns of wire along the core makes little difference, just so the turns all pass around the core at some point.

Chapter 30

FORCES AND TORQUES ON MAGNETS

300. The action of a magnetic field on a magnet is the result of many small forces acting on the individual amperian currents distributed throughout the magnet. The effect of these forces is the same as if the field exerted just two forces on the magnet at two different points which have been called the north and the south poles. The concept of a magnetic pole is thus similar to the concept of a center of gravity in mechanics. There the action of gravity really consists of separate forces acting on the distributed particles of a body, but the body behaves as if there was a single downward force at the center of gravity.

301. Magnetic Pole Strength. If two magnetic poles are placed in the same field, one may experience a stronger force than the other. The strength of a pole p is accordingly defined as the amount of force F acting on the pole per unit field H, so that

$$p = F/H \tag{301-1}$$

If the same pole is placed in any other field, the force is found to be proportional to the field, and can be computed by writing the above equation in the form $F = pH$. It follows from Eq. (301-1) that the mks unit of pole strength is a newt/(amp/m). This is the same unit as the weber which is used for measuring magnetic flux.

In a bar magnet with one pole at each end, the two poles are always opposite in sign but equal in magnitude. Thus the resultant force of a uniform field on both poles is always zero.

302. Torque on a Magnet in a Field. The torque which tends to align a magnet with a uniform field H may be analyzed in terms of the forces acting on the separate poles. The force F

on the north pole will be pH, and it will act in the direction of the field. The force on the south pole will be equal and opposite. As indicated in Fig. 302 each force will produce a counter-clockwise torque about the center equal to Fa, where a is the lever arm as shown. The total torque L will thus be given by

$$L = 2\,Fa = 2\,pHa \tag{302-1}$$

If ℓ is the length of the magnet between the two poles, then $a = (1/2)\,\ell \sin\theta$ where θ is the angle between the field and the axis of the magnet. Using ℓ the torque may be expressed in the form

$$L = p\ell H \sin\theta \tag{302-2}$$

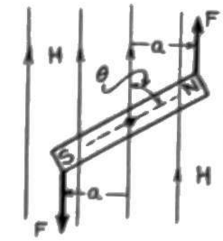


Fig. 302

If a magnet mounted on a vertical axis is acted upon by a field which is not horizontal, the vertical component of this field can produce no torque about the vertical axis. In that case, the H of Eq. (302-2) refers to the horizontal component of the total field.

In a uniform field, a torque acts as explained above, but the resultant force on the whole magnet is zero as stated in the preceding section. If the field is not uniform, the forces on the two poles may not be equal and opposite. The resultant force on the magnet in such a case will tend to pull the magnet toward a place where the field is stronger, after the magnet has once been aligned with the field.

303. Oscillations of Suspended Magnets. If a magnet mounted to turn about a vertical axis is given an angular displacement and then released, the restoring torque for any angle θ is given by Eq. (302-2) above as $L = Hp\ell \sin\theta$. If the angle of displacement is so small that $\sin\theta$ is approximately equal θ, we may write

$$L = (Hp\ell)\,\theta\,. \tag{303-1}$$

Since the restoring torque is then approximately proportional to the angle, the resulting angular motion will be approximately simple harmonic, with a frequency f given by

$$f = \frac{1}{2\pi}\sqrt{\alpha/\theta} = \frac{1}{2\pi}\sqrt{Hp\ell/I} \tag{303-2}$$

Here I is the moment of inertia of the magnet about the axis of rotation. Thus the frequency of oscillation will be proportional to the square root of the magnetic field at any point, in the same way as the frequency of a pendulum is proportional to the square root of the gravitational pull at any point.

304. Forces between Magnetic Poles. It is impossible in practice to have an isolated pole of either kind, or to ascribe a single geometrical point as the location of a pole which occupies a considerable volume. However, if we consider a pole at the end of a long, thin magnet we may assume that the other pole is so far away in empty space that no appreciable force will act on it. On this assumption it can be shown (See Appendix D) that two concentrated magnetic poles p_1 and p_2 a distance r apart will exert a force F on each other according to the equation

$$F = \frac{1}{4\pi\mu_0}\,\frac{p_1\,p_2}{r^2} \tag{304}$$

The mutual force will be either a repulsion or an attraction, depending on whether the poles are like or opposite. This equation can be approximately verified by making measurements with poles at the ends of long magnets.

305. Field due to a Magnetic Pole. For a situation where Eq. (304) applies, we may rewrite that equation to give

$$\frac{F}{p_1} = \frac{1}{4\pi\mu_0}\,\frac{p_2}{r^2} \tag{305}$$

Since F/p_1 is the field H acting on p_1, and since this field comes from p_2, we may write

$$H = \frac{1}{4\pi\mu_0} \frac{p_2}{r^2}$$

for the field due to the pole p_2 at a point a distance r away.

Chapter 31

GENERATORS AND MOTORS

310. Emf Induced in a Rotating Rectangular Coil. If a flat rectangular coil PQST as shown in Fig. 310-1 is rotated in a uniform magnetic field B, an emf will be induced in the coil since the rotation causes a change in the flux through the coil. The position of the coil in Fig. 310-1 is taken as the zero position, and Fig. 310-2 shows the coil just after it has passed through the zero position rotating counter clockwise with a constant angular velocity ω. The angle θ for the

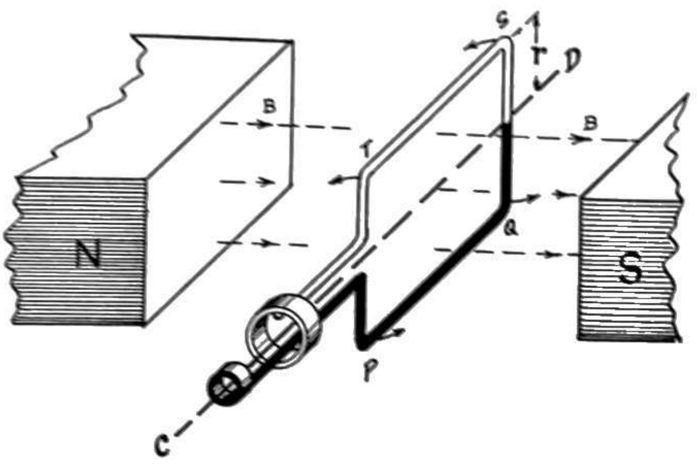


Fig. 310-1

instant represented in Fig. 310-2 will therefore be equal to ωt. As the coil rotates, the side ST moves sidewise around in a circular path of radius r with a tangential velocity as indicated by the vector V_0 in Fig. 310-2. V_0 may be resolved into rectangular components, and the component V which is perpendicular to the field is the component which is effective in producing a motion emf according to the equation E = BLv of Sect. 257. Now $V = V_0 \sin\phi$, and the angle ϕ is equal to the angle θ. Also $V_0 = \omega r$ where r is the distance from the axis of rotation to the side ST, as shown in Fig. 310-1. Hence the emf e induced in the wire ST at any particular instant may be written

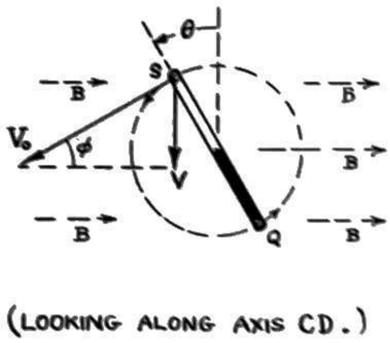


Fig. 310-2

$$e = BLr\,\omega \sin\theta \tag{310-1}$$

The emf in the side ST will be from S to T as the top side moves down. At the same time the side PQ is moving up and an equal emf will be induced from P to Q. Thus the two emfs in the two sides act around the coil in the same direction, and the total emf in the coil around from P, through Q and S to T will be

$$e = 2BLr\,\omega \sin\theta\,. \tag{310-2}$$

Since L x 2r is the area A of the coil, and since $\theta = \omega t$, we may write

$$e = BA\,\omega \sin\omega t. \tag{310-3}$$

It follows that the emf at any point is proportional to the angular velocity ω. Note that ω must be expressed in terms of radians for Eq. (310-2) to hold. The emf will have a maximum value equal to $BA\omega$ when the angle ωt is 90° or 270°. At those times the wires are moving straight up or down through the field B, and "cutting" the lines of flux most rapidly. The emf will be zero when the angle ωt is 0 or 180°, because then the wires are moving horizontally without "cutting" any lines of flux.

311. Emf Induced in Any Rotating Coil. An expression for the emf induced in any coil rotating in a magnetic field can be derived by calculus from the general equation for induced emf, $E = -N d\Phi/dt$. Consider a flat coil as shown in Fig. 311, arbitrarily choosing a positive direction around the coil as indicated on the figure. Let JK be a vector perpendicular to the plane of the coil that is related to the positive direction around the coil by the right hand screw relation. If the coil is rotated with an angular velocity ω as indicated about an axis which is perpendicular to the vector JK and the field B, the angular position at any time t can be represented by an angle θ. In general, the flux Φ through any flat coil in a uniform field B is equal to the value of B times the effective area of the coil as seen looking along the lines of flux. If A is the area of the coil in Fig. 311, the effective area as seen looking along B at any instant will be $A\cos\theta$. Hence we may write

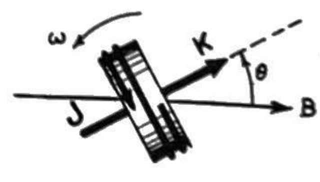


Fig. 311

$$\Phi = BA\cos\theta \tag{311-1}$$

If we start counting the time when $\theta = 0$, then the value of θ at any time t will be ωt, so that

$$\Phi = BA\cos\omega t \tag{311-2}$$

The emf e at any particular instant is $-N\,d\Phi/dt$, so that we can write

$$e = \omega NBA \sin\omega t \tag{311-3}$$

This is in agreement with Eq. (310-3) of the preceding section. That equation gave the magnitude of e for a coil of one turn where N = 1.

Note that the magnitude of the flux through the coil will be a maximum (= BA) when θ is 0° or 180°, while the magnitude of the rate of change of the flux will be greatest when θ is 90° or 270°. If Φ_0 is the maximum value of the flux for any position, we may write

$$e = N\omega\Phi_0 \sin\omega t. \tag{311-4}$$

312. Commercial Generators. The rotation of a coil in a magnetic field to produce an emf finds many important applications. Mechanical devices for the production of an emf in this way are commonly referred to as electrical "generators." The essential parts of such a generator include the rotating coil, a "field" magnet to produce the magnetic field, and some way of making

electrical connections from the terminals of the rotating coil to stationary external connections. The rotating coil generally has an iron core to increase the flux through it, and it is referred to as the "armature." The field magnet is usually an electromagnet, and in some generators, current from the armature may be used to produce the field. Such generators are said to be "self-exciting." Large generators are sometimes constructed so that the armature is fixed and the field magnet rotates, producing a changing flux in the armature coils, but our discussion of generators will be limited here to those with a rotating armature.

A generator is essentially a device to convert mechanical power into electrical power. It furnishes electrical energy by driving a current through a conducting path connected to its terminals. This path is referred to as the load of the generator. The power expended depends on the current that the load takes from the generator. The mechanical power required to keep the generator running also depends on this current. This is true because the larger the current taken from the armature, the larger will be the force opposing the rotation of the armature in the magnetic field. Thus it is that the power required to keep the generator running is determined by the amount of electrical power taken by the load. This must necessarily be true according to the principle of the Conservation of Energy. It follows from above that if no current flows in a generator, no mechanical power will be required to keep it turning except a relatively small amount needed to overcome friction and to provide for the hysteresis energy loss in the core of the armature.

313. Alternating Current Generators. The ends of a rotating coil in a generator may be connected to stationary external terminals by providing each end of the coil with a so-called "slip-ring." These are conducting rings mounted on the armature shaft but insulated from it as illustrated in Fig. 310-1. Two stationary "brushes" are then placed to give constant sliding contact with the respective slip rings, and the two external terminals are attached to these two stationary brushes. If an external path or load is connected to the terminals of such a generator, the current flowing in the load will be an alternating current. This current will change its direction back and forth in a cyclic fashion according to the alternations in the emf. The abbreviation a.c. and d.c. are commonly used to designate alternating and direct currents respectively.

The emf e generated at any instant in the rotating coil of an alternating current generator is a sinusoidal emf as given in Eq. (310-3). Using E_0 to represent the maximum value ($BA\omega$) of e, we may write

$$e = E_0 \sin\omega t$$

A graph of e vs t is shown in Fig. 313, placed below a series of drawings showing the position of the rotating coil at various corresponding times.

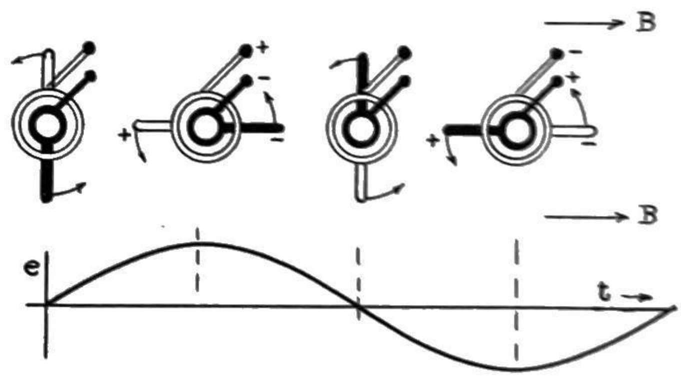


Fig. 313

If the field of an alternating current generator is furnished by an electromagnet, this magnet must be excited by some separate source of direct current.

314. Direct Current Generators. If the two external terminals of a rotating coil are interchanged relative to the ends of the coil each time the emf changes in direction, then the emf in the coil will

always be directed toward the same external terminal. Thus one external terminal will always be positive and the other one will always be negative. Also the current through the external load will always be in the same direction.

The action of a so-called commutator that automatically interchanges the connection of two terminals to the ends of a rotating coil at regular intervals is shown in the sequence of four drawings in Fig. 314. This commutator consists of a metal ring which has been cut into two semi-circular segments as shown, and mounted on the armature axle with the two halves insulated from the axle and from each other. The two halves of this ring are permanently connected to the two respective ends of the coil, and the commutator and coil rotate together. In these drawings, the plane of the coil is represented by the lines which are vertical in the first of the four drawings. The conductors shown in solid black are conductors that have negative potential. The two stationary terminals T_1 T make contact with the commutator segments through stationary sliding brushes which are shown cross-hatched. The four drawings represent the rotating coil at four successive positions a quarter of a revolution apart. Note that as the coil rotates through the position where the emf reverses, the dividing lines on the commutator ring pass under the brushes, and each brush is switched over to the opposite bar from the one it touched before. Thus a generator having such a commutator will furnish a direct current through an external load connected to its terminal.

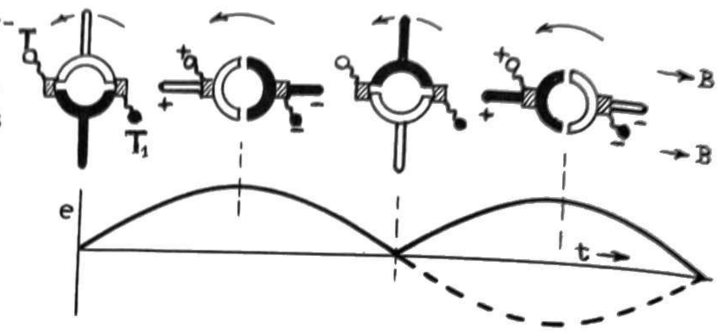


Fig. 314

The emf furnished by a direct current generator having one flat rotating coil as illustrated in Fig. 314 will always act on the load in the same direction, but its magnitude will not be constant. A graph of the emf furnished is plotted against the time for one revolution in the lower part of Fig. 314. Note that the curve is a sine curve with the negative loop reversed into a positive loop.

In order to furnish a more uniform emf than that produced by one flat rotating coil, direct current generators usually have armatures with a number of flat coils mounted on the same core and connected together in series. These coils are placed at different angles around the axis of rotation so that at any instant there will be at least one coil passing through the angular region where the induced emf is largest. By using a commutator ring which has as many separate segments as there are junctions between coils, it can be arranged so that two brushes placed on opposite sides of the commutator will always be bridged across a number of coils all having large emfs in the same direction. In this way, the emf between the brushes is maintained rather uniformly at a high value. Such an armature is illustrated schematically in Fig. 315. The two brushes connected to the terminals AA are stationary while the coils and commutator rotate together.

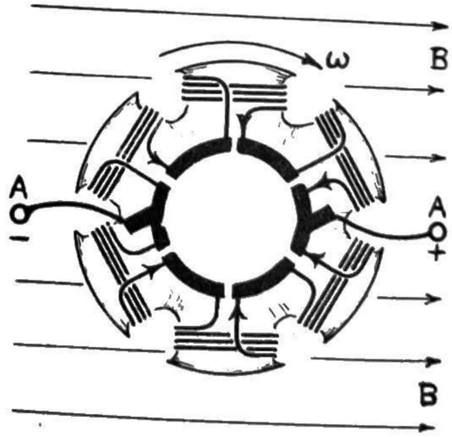


Fig. 315

315. Electric Motors. Most direct current generators can also be made to operate as electric motors. This behavior may best be explained with reference to a simple generator having one flat coil with a split ring commutator as represented in Fig. 314. If a current from some outside source is sent through such a coil, the current will make the coil turn toward a position perpendicular to the field with its

lines of flux in the direction of the field (See Sects. 203-04). Although the torque of the field on the coil becomes zero when the coil reaches a perpendicular position, its inertia will keep it moving. Rotation past this position reverses the brushes with respect to the end of the coil, and thereby reverses the current in the coil. The reversed current will then turn the coil on through 180° to the next perpendicular position, and so on.

An emf applied to a motor armature from the outside must expend electric energy to send a current through the armature. As the armature turns under the force of this current, it can do mechanical work. Thus the essential function of an electric motor is to convert electric energy into mechanical energy.

As indicated above, a given machine may operate either as a motor or as a generator. Electric locomotives have been made to operate in either manner. Going up hill, they can operate as motors to convert electrical energy into mechanical energy. Going down hill they can operate as generators and feed energy back into the power line. In general, however, the differences in practical design between generators and motors are often such that one machine will not operate efficiently for both purposes. Motor armatures are usually made with several coils in more than one plane as indicated in Fig. 315 to give a more uniform torque.

When a direct current generator is serving as a motor, its armature is rotating in a magnetic field. Since the rotation of any coil in a magnetic field will induce an emf, the armature must be generating an emf even when it is turning as a motor. Thus there will be an induced emf in the armature of a rotating motor, and our rules for direction will show that this induced emf is opposite to the current which makes the motor turn. Also, fundamental energy concepts show that an emf opposite to the current must exist in any type of motor to permit the motor to receive electrical energy and convert it into anything other than heat. Because the emf induced in a motor is always opposed to the current, it is often referred to as a "back" emf.

316. Generators and Motors in Electric Circuits. The general principles which determine the behavior of both generators and motors in direct current circuits will be treated in terms of a single machine consisting essentially of an armature rotating in a fixed field set up by an electromagnet. Let us assume that this field is maintained by a current I_F through a "field" coil having a resistance R. It will be assumed also that the field magnet is saturated so that the field is practically constant for all values of I_F which are used. The armature coils constitute a conducting path having an equivalent resistance R_A . When the armature is turning in a field, there is an induced emf E along the path through the armature. This emf will be the same whether the armature is turning as a generator or a motor. Except for some secondary effects that need not be considered here, the emf induced in the armature will depend only on the rotational speed of the armature.

A rotating armature is a path containing an emf and an internal resistance. It is therefore electrically equivalent to a voltaic cell except that mechanical energy is involved instead of chemical energy. When charges are allowed to flow through the armature in the direction of the emf, the charges gain electric energy and mechanical energy is required to keep the armature turning. Thus mechanical energy is converted into electric energy according to Eq. (156-1). When a current is sent from the outside through the armature against the induced emf, electric is converted into mechanical energy, and the machine operates as a motor according to Eq. (157). In either case, the power converted is the product of the induced emf E and the armature current I_A .

While the details of motor and generator construction will not be taken up in this course, the electrical connections in a few simple types will be studied merely as general examples of electrical networks containing resistance and emfs. The motors and generators to be considered here consist of an armature, and a field winding which may be connected either in series or in parallel with the armature, as indicated in Fig. 316. In either case, V is the difference in potential across the combination of field and armature considered as a single unit, and I is the current through the combination.

317. Efficiency of a Generator or Motor. The efficiency of a generator is equal to the ratio of the terminal output of electric power VI to the total input of mechanical power. The efficiency of a motor is the ratio of the useful output mechanical power to the input of electric power VI. The losses inside a generator or motor include mechanical losses due to friction, hysteresis loses, and electric losses due to the resistance of the armature and the field. The internal losses appear in an operating machine in the form of heat.

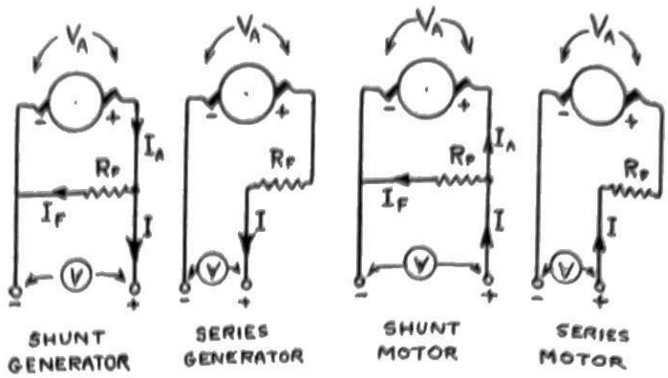


Fig. 316

Chapter 32

CURRENTS WITH CHANGING MAGNITUDES

320. The flow of charge does not always take place at a uniform rate. For example, when we close a circuit containing a constant emf, there is a short but measureable interval of time during which the current is increasing from zero to its final steady value. Also when a steady current is interrupted, there is an interval of time required for the charges to come to rest. In preceding chapters, we have considered electric currents with a constant rate of flow. In this chapter we will consider some common examples of currents that change as time goes on.

321. Discharge of a Capacitor through a Resistance. When a charged capacitor is discharged by connecting a resistive path R between its terminals, a temporary surge of current flows through the resistance from the positive terminal to the negative terminal. At any particular instant, the value of the current will be given by the equation

$$I = V/R \tag{321-1}$$

where V is value of the potential difference at that instant. Since the current is the rate at which the charge flows out of the capacitor, we may write

$$I = - dQ/dt. \tag{321-2}$$

Here Q is the charge on the capacitor at any instant and t is the time. The negative sign is used because the current is associated with a decrease in Q. Now since

$$Q = CV, \tag{321-3}$$

and since C is constant we may also write

$$\frac{dQ}{dt} = C\,\frac{dV}{dt} \tag{321-4}$$

Combining the above equations, we get

$$-\frac{dV}{dt} = \frac{V}{RC} \tag{321-5}$$

Thus the rate of decrease of the voltage at any instant depends on the voltage at that instant, and

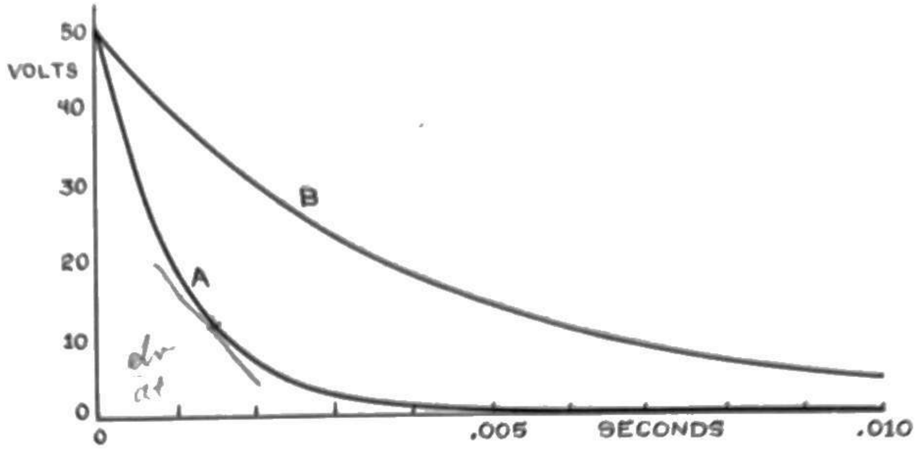


Fig. 321

it is inversely proportional to the resistance. As the discharge progresses, the voltage decreases, and the rate of discharge decreases proportionately. A graph of voltage plotted against the time will accordingly appear like the curve A of Fig. 321. This curve shows the voltage decreasing from an original value of 50 volts. The curve B shows how the same capacitor would discharge more slowly through a path having more resistance. In either case, it is to be noted that the negative slope dV/dt becomes less steep as V becomes less in accordance with Eq. (321-5). Also both curves eventually approach the t-axis asymptotically.

322. Exponential Form of the Equation for the Discharge of a Capacitor. Curves which obey an equation like Eq. (321-5) above are called exponential curves, because they can also be represented by an equation having the form

$$V = V_0 \, e^{-t/RC} \tag{322}$$

Here V_0 is a constant equal to the initial value of V. In other words, V_0 is the value of the dependent variable V when the independent variable t is zero. Equation (322) can easily be reduced to Eq. (321-5) by the differentiation process of calculus, showing that both equations represent the same relationship between V and t.

The exponential relationship found here is of considerable interest because the same mathematical equation is associated with a number of other important physical phenomena such as the absorption of radiation and the decay of radioactive materials.

323. Time Constant of a Discharging Capacitor. Looking at the discharge of a capacitor as a process in which the voltage changes from an initial value V_0 to a final value of zero, the remaining voltage V at any time represents the part of the entire process which remains to be accomplished. Thus the ratio V/V_0 is the fractional part of the process which remains to be accomplished at any time. For example, if V_0 was 100 volts, and V is 20 volts we may say that one fifth of the discharging process remains to be accomplished. The ratio of V to V_0 is involved in Eq. (322), which may be written

$$\frac{V}{V_0} = e^{-t/RC} \tag{323}$$

Thus if we allow a time t = RC to elapse, the ratio V/V_0 will be equal to e^{-1}, which is 1/2.72 or .368. This amount of time is called the time constant for that particular combination of R and C. Summarizing, we may say that after an interval of time equal to the time constant RC has elapsed, approximately one third of the original exponential process remains to be completed.

The time needed for an exponential process to be carried out to any desired degree can be computed by using Eq. 323. For example, if we wish to discharge a capacitor until only one one-thousandth of the original voltage remains, we must discharge it for a time interval to such that $e^{-t/RC} = .001$. From exponential tables we can find that t/RC must be equal to seven. In other words, t must be equal to seven times the value of the time constant RC.

A capacitor may be charged from zero voltage to a voltage V_0 by connecting it through a

resistance R to a cell furnishing a fixed voltage V_0. In this case, the increase in voltage will follow the same pattern with respect to time as does the decrease in voltage when the capacitor is being discharged. For example, a fraction .368 of the whole charging process will remain to be accomplished when t = RC.

324. Current from a Capacitor. If a capacitor is charged to a given potential difference V and then discharged through a galvanometer, a fixed amount of charge will pass through the galvanometer. By alternately charging the capacitor from a battery and discharging it through a galvanometer, the capacitor can be used to send one body of charge after another through the galvanometer. Such an intermittent flow of charge can be regarded as a pulsating current having an average value equal to the amount of charge passing through the galvanometer per second.

A current obtained by repeatedly discharging a known capacitance through a galvanometer can be used to calibrate the galvanometer. Also a previously calibrated galvanometer can be used to measure an unknown capacitance in the same way. A circuit as shown in Fig. 324 can be used in either case. The capacitor C is charged from the battery B by moving the two-way key to F; then the capacitor can be discharged through the galvanometer G by moving the key to E. Successive discharges sent through the galvanometer by moving the key back and forth will tend to deflect the coil with intermittent impulses, but the inertia of the coil will prevent it from following the pulsations of the current in complete detail. If the pulsations follow each other closely enough, the galvanometer deflection will oscillate only slightly about an intermediate position which indicates the average value of the current.

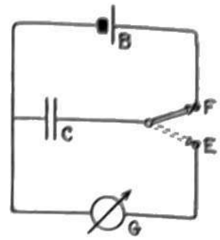


Fig. 324

The relationship between the average current through the galvanometer and the other quantities involved may be derived as follows. Let V stand for the potential to which the capacitance C is charged. The charge Q which is discharged through the galvanometer each time will then be equal to CV. Now the amount of charge passing through the galvanometer in one second will be equal to the number of discharges in one second multiplied by the charge Q in each discharge. Hence if n is the frequency with which the key is operated back and forth, the average current I will be nQ, so that

$$I = nCV, \qquad (324\text{-}1)$$

or

$$C = I/nV. \qquad (324\text{-}2)$$

In practice the key V may be operated by an electrically driven tuning fork, or by a constant speed motor. For most suspended-coil galvanometers, a frequency of several discharges per second is enough to give a noticeably steady deflection. There is an upper limit to the frequency with which the key can be operated because the key must remain closed in each direction long enough to give the capacitor time to charge or discharge.

325. Mutual Induction. A changing current in a circuit will change the magnetic flux in the surrounding space, and this changing flux may induce an emf in another nearby circuit. Such induction occurs even when there is no electrical connection between the circuits. It is known as mutual induction, and it is one example of the general phenomena of induced emf as discussed in Chap. 25.

When mutual induction occurs, the circuit in which the changing current flows is called the "primary" circuit; The circuit in which the emf is induced is called the "secondary" circuit. The magnitude of the emf E_2 induced in the secondary circuit depends on the rate of change of the flux through that circuit and hence on the rate of change of the current I_1 in the primary circuit. This dependence may be expressed by an equation having the form

$$E_2 = -M(dI_1/dt). \qquad (325\text{-}1)$$

Here M is a factor depending on the geometry of the two circuits and on the presence of any magnetic core material. This factor M is called the coefficient of mutual inductance between

the two circuits. The negative sign is used to indicate that E_2 and I_1 are in opposite directions when I_1 is increasing, provided the two circuits are placed like two coils wound in the same direction on the same straight core.

Equation (325-1) can be written in the form

$$M = \frac{-E_2}{(dI_1/dt)} . \tag{325-2}$$

This shows that the coefficient of mutual inductance may be defined as the emf induced in the secondary circuit per unit rate of change of current in the primary circuit. The most common unit of mutual inductance is a volt per ampere per second; This is more briefly referred to as a henry. For example, if an emf of 100 volts is induced in a secondary circuit when the current in the primary circuit changes at the rate of 20 amperes per second, the mutual inductance between the two circuits is 5 henrys. A millihenry is 10^{-3} henry.

326. Mutual Inductance between Coils. Some of the factors which determine the value of the mutual inductance between two coils may be pointed out with reference to Fig. 326. In general, the amount of flux through the secondary will be proportional to the current I_1 in the primary. It will also depend on the number of turns N_1 in the primary and on the relative position of the two coils. For a given amount of flux from the primary coil, the emf induced in the secondary will depend on the number of turns N_2 in the secondary. It follows that the mutual inductance between two coils as shown in Fig. 326 could be increased by increasing the number of turns in either coil, by moving the two coils closer together, by increasing the area of the coils, or by turning the two coils into a more favorable orientation with respect to each other.

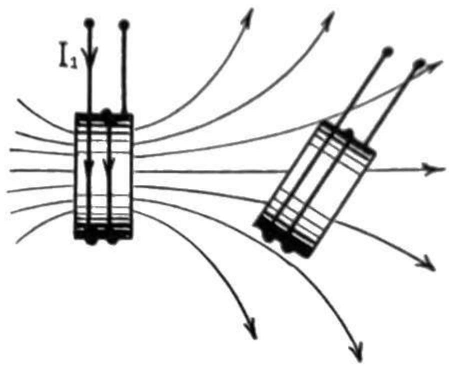


Fig. 326

The mutual inductance between two coils can be greatly increased by using a ferromagnetic core which extends through both coils. A given change of primary current will then produce a larger change of flux in the secondary, provided the core is not already saturated.

For any given arrangement of two coils with air cores, the value of M will be constant for all values of the primary current. This is true because the flux density at any particular point is strictly proportioned to the primary current. If a ferromagnetic core is used to obtain a larger value of M, the flux will not be strictly proportional to the primary current and the value of M may be different for different primary currents.

327. Self-inductance. When a current I_1 flows in a circuit, there will in general be magnetic flux through the circuit due to this current. If the current changes, the flux through the circuit will change. This will induce an emf around in the circuit that carries the current, just as it will in any other circuit through which the flux passes. The induction of an emf in the circuit that carries the changing current is called self-induction.

The self-induced emf E_1 in a circuit depends on the rate of change of the current I_1 in the same circuit, and thus dependence can be expressed by the equation

$$E_1 = - L(dI_1/dt). \tag{327-1}$$

Here L is a factor depending on the geometry of the circuit, and it is called the coefficient of self-inductance. The negative sign in the equation indicates that E_1 and I_1 are opposed to each other when I_1 is increasing. When I_1 is decreasing, dI_1/dt will be negative, and E_1 and I_1 will be

in the same direction. These directional relationships are consistent with the general rules already given for the direction of any induced emf.

By writing Eq. (327-1) in the form

$$L = -\frac{E_1}{(dI_1/dt)} \tag{327-2}$$

we see that the coefficient of self-inductance is defined as the amount of self-induced emf per unit rate of change of current. Eq. (327-2) also shows that a volt per ampere per second (or henry) can be used as a unit for measuring self-inductance as well as for mutual inductance. A circuit will thus have a self-inductance of one henry if an emf of one volt is induced in the circuit when the current changes at the rate of one ampere per second.

328. Self-inductance of a Coil. Any circuit or any part of a circuit will exhibit a certain amount of self-inductance. The property of self-inductance is most marked however, in a conducting path such as a coil where adjacent turns of wire carry current in the same direction. The way in which the self-inductance is increased by adjacent turns may be seen by considering a flat circular coil in which all the turns have practically the same diameter. In that case, the self-inductance L will be approximately proportional to the square of the number of turns. For example, doubling the number of turns through which the current I_1 flows will double the amount of flux; the induced emf will then be doubled again because this doubled flux passes through twice as many turns.

If desired, the self-inductance of a long piece of coiled wire can be reduced by a system of duplex winding so that wherever there is a current flowing in one direction there will also be a current flowing in the opposite direction in a nearby parallel wire. Thus any field produced by one wire of a pair will be neutralized by the opposite current in the other wire. If there is no resultant field, there can be no flux to induce an emf.

The self-inductance of a coil can be increased by use of a ferromagnetic core which will give a larger change in flux for a given change in current. To get the full benefit of the core, the current values should not exceed the current required to saturate the core. If the core is saturated, changes in current will produce no change in the magnetization of the core, and the core will have no effect. Material used for cores should be easy to magnetize and demagnetize so that its magnetization will follow closely the changes in current.

329. Energy of a Current. If we wish to establish a current in a path having self-inductance, we must expend energy. The current must be increased against the self-induced emf which always opposes an increasing current. If we wish to stop a current which is already flowing, the induced emf will act in the same direction as the current and tend to keep the current flowing. Thus self-inductance in a path makes the current behave as if it had an electric inertia analogous to the inertia of a material body. To start the motion in either case, we must apply a certain amount of energy. Also in either case, this energy reappears again when we stop the motion. In the case of mechanical motion, we say that the energy expended to start the body is stored in the moving body as kinetic energy. In the case of a current, we conclude that the energy expended to start the current is stored in the magnetic field of the current, since the induced emf depends upon the magnetic field.

The amount of energy stored when a current flows in an inductive path may be found by computing the energy that is made available when the current is stopped. If a current I is stopped at a uniform rate in a time t, there will be a constant emf E in the path during this time. This emf will be equal to $L(dI/dt)$ and it will act in the direction of the current. Thus the induced emf will furnish energy to the current. The energy W expended by the induced emf in the path will be equal to $E\bar{I}t$, where $\bar{I}$ is the average current during the time t. For a uniform rate of decrease, $\bar{I} = I/2$ and $dI/dt = I/t$, so that

$$W = LI^2/2. \tag{329}$$

The striking analogy between self-inductance and mechanical inertia may be seen by comparing a number of equations in mechanics and in electricity. For example, we see that the mechanical equations listed in the first column below have the same forms as the respective electric equations in the second column if we take L to be analogous to the mass m, I to be analogous to the velocity v, and E to be analogous to the force F.

$F = m(dv/dt)$	$E = -L(dI/dt)$
$\text{Energy} = Fvt$	$\text{Energy} = EIt$
$\text{K.E.} = mv^2/2$	$\text{Energy} = LI^2/2$

Chapter 33

CURRENTS WITH CHANGING MAGNITUDES (continued)

330. Growth and Decay of Currents in Circuits Having Self-inductance. If an emf E is applied to a path having a total resistance R and a self-inductance L, there will also be an induced emf E_i equal to $- L(dI/dt)$ whenever the current is changing. The emf E_L that must be applied to overcome E_i will be opposite to E_i and therefore equal to $+ L(dI/dt)$. The emf E_R that must be applied to overcome the resistance will be IR. Hence the total emf E that must be applied will be $E_L + E_R$, or

$$E = L\frac{dI}{dt} + IR \qquad (330\text{-}1)$$

This equation may be written in the form

$$L\frac{dI}{dt} = E - IR \qquad (330\text{-}2)$$

to show how the current will increase at any particular time. The equation states that the voltage $L(dI/dt)$ which goes to increase the current is equal to the total applied voltage E, less the voltage IR required to maintain the existing current in the resistance. At first, before I has acquired any appreciable value, IR will be zero and all of the applied voltage will go to give a maximum initial rate of increase of current equal to

$$\frac{dI}{dt} = \frac{E}{L} \qquad \text{(when } t = 0) \qquad (330\text{-}3)$$

As the current increases, more of the applied voltage E will be used up in the IR drop, and less will be available for increasing the current. Thus dI/dt gets less as I gets greater. Finally, the current will approach asymptotically a value which uses all of the applied voltage in the IR drop, and there will be no further increase in the current. When this steady state is reached, $dI/dt = 0$ in Eq. (330-2), and hence

$$0 = E - IR \qquad \text{or} \qquad I = E/R \qquad (330\text{-}4)$$

It follows from above that a graph of I plotted against t will start from the origin with an initial slope dI/dt equal to E/L, the slope will decrease as t gets larger. For large values of t the graph will approach a straight horizontal line for which I has the constant value E/R. Two such curves are shown in Fig. 330. Note that the larger the inductance, the more slowly the current will increase for a given emf, E. Hence self inductance tends to give a circuit a sluggish response to applied emfs.

If a current I is flowing in a closed circuit of resistance R after an external emf E is

removed, Eq. (1) or (2) will apply with E made equal to zero. Thus we may write

$$\frac{dI}{dt} = -\frac{R}{L} I \qquad (330\text{-}5)$$

This equation has the same form as Eq. (321-5). Here we have I as the dependent variable instead of V, and L/R as a constant term instead of 1/RC. The curve of I vs t will accordingly be an exponential curve to which the equation

$$\frac{I}{I_0} = e^{-\frac{t}{L/R}} \qquad (330\text{-}6)$$

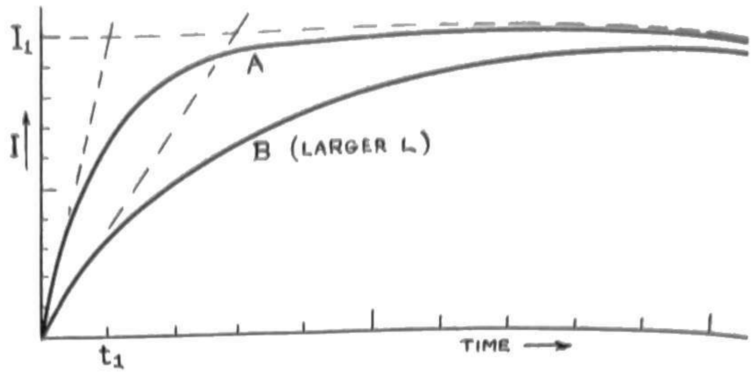


Fig. 330

applies, with I_0 being the initial value of the current when it starts to decrease. Thus L/R is the value of t for which $I = .368\, I_0$. (See Sect. 322).

331. The Induction Coil. Temporary surges of high voltage can be obtained from a relatively low voltage source by using an induction coil. One common form of induction coil shown in Fig. 331 is used to furnish the ignition spark in an automobile engine. These coils are made with a secondary coil of many turns wound over a primary coil having an iron core. The current in the primary is furnished by the six-volt storage battery. When the contact points C in the primary circuit are closed, the current I_1 builds up rather slowly, giving a relatively small induced emf in the secondary. When the contact points are opened, the primary current must stop very abruptly, giving a rapid rate of decrease in the flux through the secondary coil. This induces a relatively large emf in the secondary coil, which causes a spark to jump across the gap of a spark plug s. In motor cars, the contact points are opened and closed mechanically at the proper times to ignite the charges of gas in the respective cylinders.

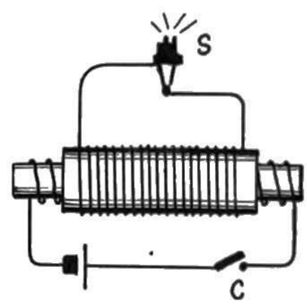


Fig. 331

Induction coils for various other purposes may be constructed so that the contact points in the primary circuit are opened and closed by a self-sustained vibration, using a mechanism like that of the common electric door bell.

Chapter 34

ALTERNATING CURRENTS

340. Alternating electric currents that change back and forth in direction are of great interest for a number of reasons. They are widely used in the transmission of electric power for industrial and domestic applications. They are involved in the electric transmission and reproduction of sound. In the form of electric oscillations, they occur as an essential feature in most of the radio and electronic devices which are so widely used today.

Although the same fundamental principles of electricity and energy apply to all currents, there are a number of special ideas and relationships that apply to alternating currents alone. We will now consider some of the outstanding characteristics of these currents.

341. Sinusoidal Emfs and Currents. A sinusoidal alternating emf e such as is induced in a rotating coil (Sect. 310) is one that can be described by the equation

$$e = E_0 \sin(\omega t + \alpha) \tag{341-1}$$

Here e is the instantaneous value of the emf at a time t, while E_0, ω, and α are constants. The meanings of these constants are indicated in Fig. 341, where a graph of Eq. (341-1) is shown. The reference circle associated with the sine curve is also shown in the figure. The sine curve

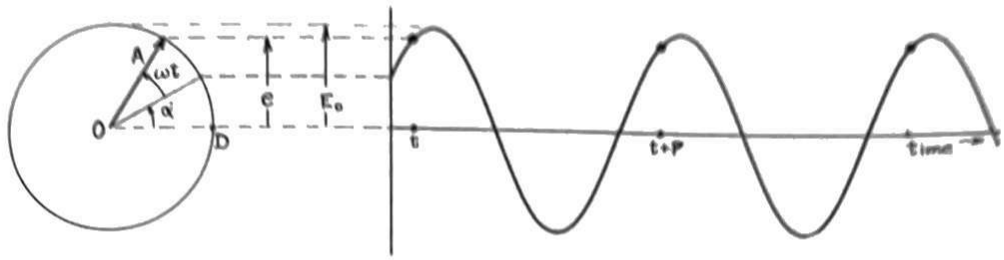


Fig. 341

is shown continuously from a time of zero to a time a little over two periods later. The position of the rotating vector is shown for one particular time t, as indicated. The constant E_0 is the maximum value of e. In other words, it is the maximum ordinate of the sine curve. E_0 is also the length of the rotating vector in the reference circle. The value of e at any time is the vertical component of the rotating vector at that time.

The angle $(\omega t + \alpha)$, of which the sine is taken in Eq. (341-1), is called the phase angle. This is also the angle that appears in the reference circle between the zero line OD and the rotating vector A. The constant ω is the angular velocity with which the vector A rotates. For an emf generated by a coil rotating in a uniform magnetic field, ω is the same as the angular velocity of the rotating coil. The angle α is the value of the phase angle when we start counting time, that is when $t = 0$. The angle added in any subsequent time t will then be ωt, so that the total angle at the time t will be $\omega t + \alpha$ as shown in Fig. 341. For any given sinusoidal quantity, we can arbitrarily make α equal to zero by starting to count time when the whole phase angle for that quantity is zero. Mathematically this means that if $t = 0$ when $(\omega t + \alpha) = 0$, then $\alpha = 0$.

The sinusoidal changes associated with one complete revolution of the reference vector is called a cycle. The period P is the time required for one cycle. Since there are 2π radians in one revolution, it follows that

$$\omega = 2\pi/P \tag{341-2}$$

The frequency f of an alternating voltage is the number of cycles per second. Thus we may write $f = 1/P$ and

$$\omega = 2\pi f \tag{341-3}$$

The graph and reference circle for an alternating voltage in Fig. 341 can be adapted to represent an alternating current by merely marking the vertical scale to represent current instead of voltage. Equation (341-1) can also be adapted to represent a sinusoidal current by writing i and I_0 for e and E_0 respectively.

342. Poly-phase Alternating Currents. An alternating current in a closed circuit which obeys the equation $i = I_0 \sin(\omega t + \alpha)$ is referred to as a single-phase current. Some applications of alternating currents involve several simultaneous currents with the same frequency but with different phase. For example, some applications use two currents with one 90° ahead of the other in phase. Such currents can be obtained from a generator that has two separate rotating coils mounted on the same shaft with one coil rotating 90° behind the other. If we start counting

time so that the emf in the first coil is given by $e_1 = E_0 \sin(\omega t + 0)$, then the emf e_2 in the second coil will be $e_2 = E_0 \sin(\omega t - 90^\circ)$. Two-phase generators like this generally use a separate pair of wires for each coil, making a total of four wires. The two circuits are thus electrically separate unless they are connected together at the output end.

Three phase generators are made with three similar coils mounted on the same shaft, each coil being 120° behind the preceding one in phase. To carry the output of such a generator by completely separate circuits requires three pairs of wires. To save wire in practice, a single return wire may be used for all three circuits, so that three circuits may be had with only four wires. If the loads in the three circuits are balanced, the net current in the return wire will be zero because one will neutralize the other two, and hence the fourth wire is not necessary. In that case, the output of a three-phase generator can be carried with only three wires.

Since poly-phase generators consist essentially of two or more single phase generators mounted on the same shaft, the same fundamental alternating circuit theory will apply to all except for differences in detail.

343. Alternating Current in a Resistive Path. An alternating emf applied to some paths gives a current that is strictly proportional to the voltage at every instant, with the same proportionality factor that applies for constant currents. Such a path is said to be a purely resistive path having a resistance R given by the equation

$$R = e/i. \tag{1}$$

In a resistive path, the current must increase and decrease along with the emf. Hence if the emf is sinusoidal, the current will be sinusoidal and it will be in phase with the emf. Stated mathematically, if $e = E_0 \sin\omega t$, then $i = I_0 \sin\omega t$. Note that since $e/i = R$ for any simultaneous pair of values of e and i, and since e and i both have their maximum value at the same time,

$$R = E_0/I_0. \tag{2}$$

344. Power Expended in a Resistive Path. The amount of power P expended by an emf driving an alternating current through a resistive path will vary from one instant to the next according to the changing value of the current. This power will be expended in the form of heat. The general equation

$$P = i^2 R$$

holds at every instant for such a path whether the current is constant or changing. Since heat is produced by a current without regard to its direction, an alternating current will produce heat in pulses. There will be a "peak" of heat production whenever the current reaches its maximum in either direction, so that the frequency of the heat pulsations will be twice the frequency of the alternating current. For example, a 60 cycle current furnished by a commercial power line will give 120 pulses of heat each second. When such a current flows through an ordinary incandescent lamp, the thermal inertia of the filament keeps the fluctuation of the temperature so small that it can hardly be noticed.

A graph of the rate of heat production plotted over a complete cycle may be made by plotting values of $i^2 R$ against the phase angle. Since $i^2 R = I_0^2 R \sin^2 \omega t$, where I_0 and R are constants, this curve will be like a curve of $\sin^2 \omega t$ except for the marking of the scale along the ordinate. A graph of $\sin^2 \omega t$ over a complete cycle is shown in Fig. 344 with the corresponding current curve shown for comparison. The curve for

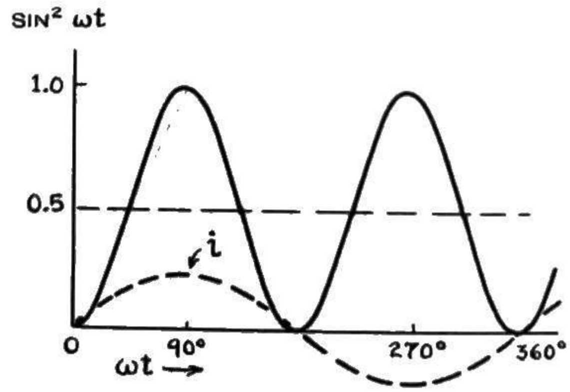


Fig. 344

$\sin^2 \omega t$ can be made by using the squared values of sines of angles from 0° to 360° as found in mathematical tables. The same graph can also be made with less labor by noting that

$$\sin^2 \omega t = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$$

Thus a simple cosine curve of twice the angle plotted on an axis 1/2 of a unit above the axis of the figure will give the same result.

345. Average Rate of Heat Production. The rate of heat production in a resistive path will have its maximum value of $I_0^2 R$ when the current has its maximum value of I_0 in either direction. The average rate of heat production will be considerably less than this amount, and will be equal to the value of $i^2 R$ averaged over any whole number of periods. The relationship between the average rate of heat production and the maximum rate can be found mathematically. To do this we may write

$$\text{Avg. value of } i^2 R = \text{Avg. value of } (I_0^2 R \sin^2 \omega t) \tag{345-1}$$

$$= I_0^2 R \times \text{Avg. value of } (\sin^2 \omega t) \tag{345-2}$$

$$= I_0^2 R \times \text{Avg. value of } \left(\frac{1}{2} - \frac{1}{2} \cos 2\omega t\right) \tag{345-3}$$

$$= I_0^2 R/2 - I_0^2 R \times \text{Avg. value of } (\cos 2\omega t)/2 \tag{345-4}$$

$$= I_0^2 R/2 \tag{345-5}$$

The last step follows from the fact that the average value of the cosine over any whole number of cycles is zero.

346. The Effective Value of an Alternating Current or Voltage. The effective value I of an alternating current is the value of a constant current that will produce heat at the same rate in a given resistance, assuming that the rate for the alternating current is averaged over a whole number of cycles. In other words, we can write

$$I^2 R = \text{avg. value of } i^2 R. \tag{346-1}$$

It follows from Eq. (345-5) that

$$I^2 R = I_0^2 R/2 \tag{346-2}$$

or

$$I^2 = I_0^2/2. \tag{346-3}$$

Hence

$$I = .707\, I_0 \tag{346-4}$$

The cancellation of R in Eq. (346-1) gives

$$I = \sqrt{\text{avg. value of } i^2} \tag{346-5}$$

This states that the effective value I of a sinusoidal current is the square root of the average (or mean) value of the squares of the instantaneous values. For this reason, the effective current is sometimes referred to as the root-mean-square current. It should be noted that this root-mean-square value is not the same as the average value of the unsquared current. The average value of the unsquared current is equal to the total charge transported divided by the time, and the total charge transported by an alternating current in any number of complete cycles is always zero.

The effective value E of an alternating emf is the value of a constant emf which could be applied to a resistance R to produce heat at the same average rate in a given resistance, assuming that the alternating emf is applied for any whole number of cycles. Thus $E = IR$, where I is the effective value of the current which flows when the a.c. emf is applied. Since $E_0 = I_0 R$, it follows from Eq. (346-4) that

$$E = .707 E_0$$

It is the universal practice to measure and specify alternating emfs and currents in terms of their effective values. Thus, an a.c. emf of 110 volts means that 110 volts is the effective value unless definitely specified otherwise. For this reason, we use the symbols E and I without any subscript to represent the effective values.

347. Alternating Currents in Inductive Paths. An alternating current flowing through a path with inductance requires an alternating emf to produce the changes in the current. This emf is in addition to the emf iR that is required to maintain a current through the resistance of the path. The difference between the two emfs may be illustrated by a mechanical analogy. The emf required to overcome inductance is analogous to the force that is required to overcome inertia in moving a heavy body back and forth along a path. The emf required to maintain a current through a resistance is analogous to the force required to overcome friction in moving a body along a path.

In general, an electric path will have some resistance and some inductance, but in this section we will consider the flow of current through an ideal path which has inductance but no resistance. This is analogous to considering the motion of a body along an ideal path where there is no friction, so that the only force required will be that needed to overcome the inertia in starting and stopping. A close practical approximation to such an ideal electric path can be had in the form of a large coil of many turns, using copper wire to keep the resistance small.

In a purely inductive path, the applied emf must be equal and opposite to the self-induced emf in the path. The self-induced emf is - L di/dt, and hence the applied emf e will be given by

$$e = L(di/dt) \tag{347-1}$$

The absence of the negative sign in this equation means that the applied emf must be in the same direction as the current to produce an increase in the magnitude of the current.

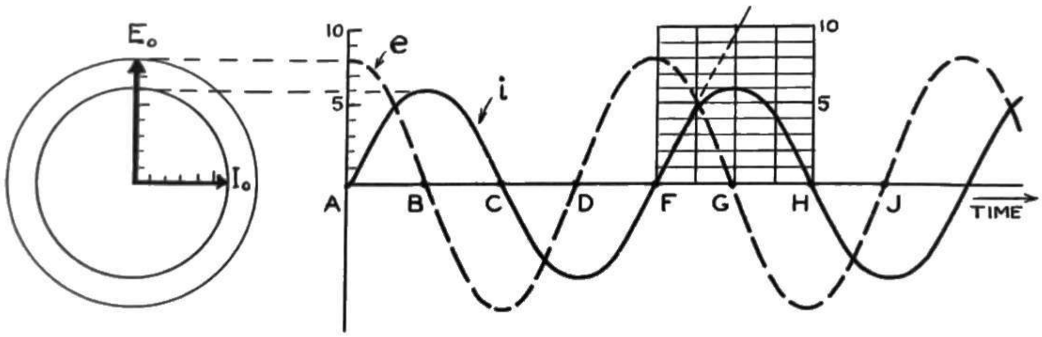


Fig. 347

To find the relationship between the emf and current when a sinusoidal current flows in a path of inductance L and negligible resistance, let us draw a sine curve to represent the current plotted against time. Whenever the value of the current is increasing, there must be an applied emf in the positive direction, equal to L times the rate of increase of the current. For example, there must be a positive emf between D and G on the figure. This emf will have a maximum value at F, since the current curve is rising most rapidly there, and will taper off to zero at G, where the current curve is momentarily horizontal. Whenever the current curve is decreasing, as between G and J, the emf must have a negative value, with its maximum negative value at H where the downward slope of the current curve is greatest. Thus the complete curve for the applied emf as shown by the broken-line graph could be drawn by making its ordinate at any point equal to L times the slope of the current curve at that point.

It can be shown by calculus that in order to produce a current having the form

$$i = I_0 \sin\omega t, \tag{347-2}$$

the applied emf e must obey the equation

$$e = E_0 \sin(\omega t + 90^\circ) \tag{347-3}$$

where the maximum emf E_0 is given by the equation

$$E_0 = I_0 \omega L\ ^{1} \tag{347-4}$$

To show what it means for the emf to have a phase angle $(\omega t + 90^\circ)$ while the current has an angle ωt, consider both angles at a time when $t = 0$. At that time the phase angle of the current is 0° while that of the emf is 90°. This is shown on the reference circle for $t = 0$ in Fig. 347. As time goes on, both angles increase at the same rate ω, and hence the two vectors in the reference circle will rotate together with the voltage vector always 90° ahead of the current vector. Such a relationship is commonly described by saying that the emf is 90° "ahead" of the current with regard to phase. It may also be described by saying that the emf "leads" the current by 90°, or that the current "lags" behind the emf by 90°.

The significance of the difference in phase between the emf and current may also be observed by again noting the curves of Fig. 347. There the curve for e comes to a "peak" at an earlier (i.e., smaller) time than does the curve for i. This difference in time is one quarter of a period, which corresponds to a difference in phase angle of 90°.

348. Alternating Currents with Capacitors. An alternating emf applied to a capacitor, will give repeated cycles of charge and discharge in opposite directions. For an ideal capacitor, any charge placed on one terminal requires that an equal charge must be removed from the other terminal, as indicated in Fig. 348-1. Thus it appears from the outside as if the charges flow "through" the capacitor when it is being charged or discharged. It is therefore common practice to treat a capacitor as a conducting path for alternating currents, although no charge actually crosses between the plates inside the condenser. However, it must be remembered that the behavior of the capacitor as a conducting path differs considerably from that of a purely resistive path, as will be pointed out in what follows.

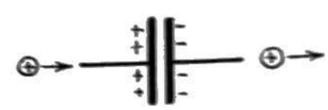


Fig. 348-1

The relationship between a sinusoidal emf applied to a capacitor and the current through the capacitor is shown by the two curves labeled e and i respectively in Fig. 348-2. The charge q on the capacitor at any instant is proportional to the voltage e at that instant, according to the fundamental relationship $q = eC$. The maximum positive current will therefore occur when the charges are flowing into the capacitor most rapidly; That is when the emf is increasing most rapidly. For example, a maximum of the current curve occurs at a time 0 when the voltage curve is crossing the axis with its steepest upward slope. There will be no current a quarter of a period later at a time A, because the voltage has stopped increasing and has not yet started to

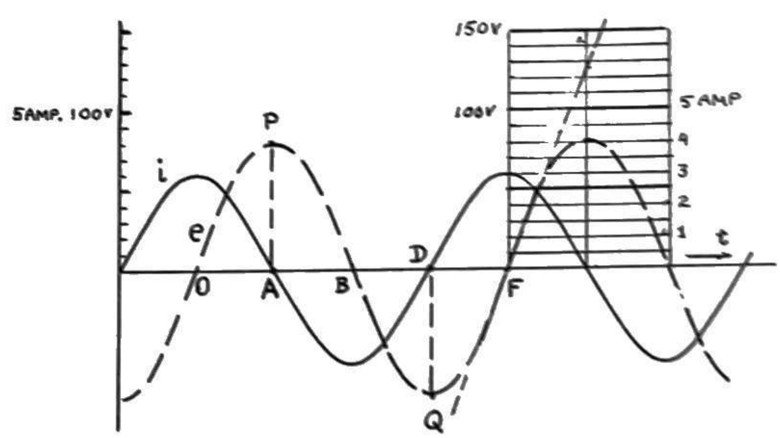
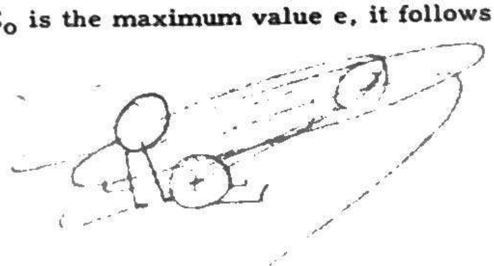


Fig. 348-2

1. If $i = I_0 \sin \omega t$, then $di/dt = \omega I_0 \cos \omega t$. Thus we may write

$$e = L\frac{di}{dt} = \omega L I_0 \sin(\omega t + 90^\circ).$$

Since E_0 is the maximum value e, it follows that $E_0 = \omega L I_0$.



decrease. Charges have stopped flowing into the capacitor, and will soon start to flow back out, giving a current in the negative direction. The negative current will reach its maximum magnitude at the time B, when the emf is decreasing most rapidly, and so on. Thus the complete current curve can be drawn in by making its ordinate at any time proportional to the slope of the voltage curve.

It can be shown by calculus that a current having the form

$$i = I_0 \sin \omega t \tag{348-1}$$

will result if a capacitor is subjected to a sinusoidal emf having the form

$$e = E_0 \sin (\omega t - 90^\circ), \tag{348-2}$$

where the maximum value E_0 of the emf is given by

$$E_0 = I_0/\omega C.^{1} \tag{348-3}$$

The -90° which appears in the phase angle of the emf in Eq. (348-2) means that the emf is 90° behind the current in phase. Thus reference circles drawn for the current and emf at zero time appear as shown in Fig. 348-3. As time goes on these vectors rotate together with the voltage vector always 90° behind the current vector. This difference in phase appears between the two curves in Fig. 348-2, where the peak of the voltage curve occurs one quarter of a period after the peak of the current curve.

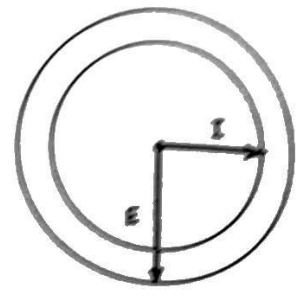


Fig. 348-3

The relationship $E_0 = I_0/\omega C$ may be written in the form $I_0 = E_0 \omega C$ to show that the current through a given capacitance due to a given emf will be proportional to the frequency. At a higher frequency, the same charge must flow in or out in a shorter time, so that the current is larger.

349. Impedance of a Path. In dealing with direct currents it was found possible to describe a simple conducting path by specifying its resistance, where the resistance was defined as the ratio of voltage to current. In order to describe an alternating current path completely, it is necessary to specify the difference in phase between the voltage and current for that path as well as the ratio of voltage magnitude to current magnitude. For example, in a resistive path the emf and current are in phase and the ratio of E_0/I_0 is R. In a path through a pure inductance, the emf is 90° ahead of the current in phase, and $E_0/I_0 = \omega L$. In a path through a capacitor, the emf is 90° behind the current in phase and $E_0/I_0 = 1/\omega C$.

The impedance of an alternating current path is a quantity defined to specify both the ratio of the effective emf to the effective current and also the angle by which the emf leads the current. Thus the impedance of a path is by definition a quantity having a magnitude Z and an angle ϕ, where Z is equal to E/I, or to E_0/I_0. We may accordingly write

$$Z = R, \text{ and } \phi = 0 \text{ for a path through a resistance} \tag{349-1}$$

$$Z = \omega L, \text{ and } \phi = +90^\circ \text{ for a path through an inductance} \tag{349-2}$$

$$Z = \frac{1}{\omega C}, \text{ and } \phi = -90^\circ \text{ for a path through a capacitor} \tag{349-3}$$

1. If $e = E_0 \sin (\omega t - 90^\circ)$, and $q = eC$, then $q = E_0 C \sin (\omega t - 90^\circ)$. From this it follows that

$$i = \frac{dq}{dt} = E_0 \omega C \cos (\omega t - 90^\circ)$$

or

$$i = E_0 \omega C \sin \omega t.$$

Since I_0 is the maximum value of i, we can write $I_0 = E_0 \omega C$ or

$$E_0 = I_0/\omega C.$$

Since Z is a ratio of emf to current, it may be expressed in the same units that are used for resistance. In fact, resistance may now be regarded as one of several types of impedance, all of which are measured in ohms. Any impedance in which the voltage is 90^0 out of phase with the current is called a reactance, and is represented by the symbol X. The type of impedance that has an angle of $+90^0$ is called a positive, or inductive, reactance. The type that has an angle of -90^0 is called a negative, or capacitive, reactance.

Since impedance has a magnitude and an angle, it can be represented by a vector. The representation of an impedance by a vector can be illustrated by considering the impedance when it has an alternating current of one ampere, and at one particular instant when the phase angle of the current is zero. The rotating vector for the current I will then be one unit long and will have an angle zero as shown in Fig. 349. Let us assume for example, that the corresponding emf has an effective value of 4 volts and that it leads the current by 60^0 as shown. This means that the impedance is 4 ohms with an angle of 60^0. We therefore see that this impedance can be represented by the same vector that represents the emf on a scale of 1 ohm for each volt. If I is not 1 ampere as assumed, the vector for E will still represent the impedance, but not on a one to one scale. For example, if I = 2 amp, then a length that represents 1 volt will represent only .5 ohm, and so on.

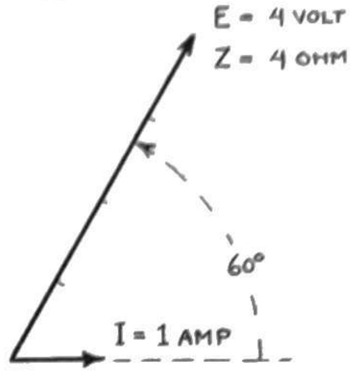


Fig. 349

As time goes on, the vector for the emf and the current will rotate together. This means that the magnitude and the angle of the impedance remain constant. It is the general practice to draw an impedance vector with its angle measured relative to a fixed zero direction extending to the right. Thus the vector representing a given impedance remains fixed as time goes on. The rotating emf vector will coincide in direction with the impedance vector only if we draw the emf vector at a time when the angle of the current vector is zero.

Chapter 35

THE MEASUREMENT OF ALTERNATING CURRENTS AND VOLTAGES

350. The measurement of alternating electric quantities generally involves different instruments from those used for direct currents. However, special meters can be made to measure alternating currents and voltages, and a.c. bridge circuits similar to the wheatstone bridge can be used to make comparative measurements. One difficulty always present in a.c. measurements is the impossibility of obtaining paths that are purely resistive, purely inductive, or purely capacitive. The characteristics of any path through any measuring circuit will therefore depend on the frequency of the alternating current. This means that instruments and techniques which are suitable at one frequency may not be good at other frequencies. In this book we will have space only to mention a few of the different types of a.c. instruments, without discussing measuring techniques in general.

351. Moving-Coil a.c. Meters. An ordinary moving coil meter cannot be used to measure alternating currents because the coil has too much inertia to follow the alternations. An alternating current flowing through a d.c. meter will produce only a slight vibration of the needle about its zero position. A moving-coil meter can be made to indicate an alternating current by using an alternating magnetic field instead of the fixed field of a permanent magnet. The construction of such an instrument is mechanically like that of a watt-meter (Sect. 241) but the electric connections are different. In an alternating current meter, the two coils are connected

in series between a single pair of binding posts. Since the alternating current through both coils reverses simultaneously, the torque on the moving coil will always act in the same direction. Thus a deflection will result which can be used as a measure of the current.

At a given frequency, the current through any a.c. meter will be proportional to the voltage between its terminals, so that the meter can be used as a voltmeter at that frequency by properly calibrating its scale to read in volts.

352. Iron Vane A.C. Meters. One common type of alternating current meter consists of a pointer with a soft iron vane attached so that the vane will be pulled toward a fixed coil of wire when a current flows in the coil. Because of its low magnetic retentivity, the vane will be pulled toward the coil regardless of the direction of the current. The magnitude of this force, acting against an elastic spring, can be used to give a measure of the current.

353. Thermo-couple A.C. Meters. An alternating current can be measured by observing the heating effect of the current in a fixed resistance. In the thermocouple type of meter, the heating effect is measured by attaching a delicate thermocouple T to a resistive path AA through which the current is made to flow, as shown in Fig. 353. A sensitive d.c. meter connected to the thermocouple is used to indicate the temperature produced by the a.c. current. The d.c. meter and the resistive path may be permanently connected together in a single unit with the scale of the d.c. meter marked to indicate directly the alternating current through the resistive path. The power expended as heat in the resistive path is proportional to the square of the current. Thus the scale of the d.c. meter must have its divisions spaced according to the square of the reading in order to give a direct indication of the alternating current.

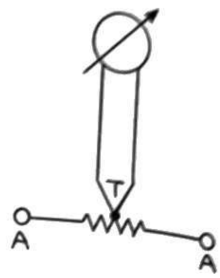


Fig. 353

Thermocouple meters for alternating current can be used over a comparatively wide range of frequencies.

354. Cathode Ray Oscillograph. The cathode ray oscillograph is one of the most useful instruments for measuring alternating currents and voltages. Among other things, it can be used to automatically draw a graph of an alternating current or voltage plotted against the time. This curve is formed on the screen of a television tube. It shows both the phase and the magnitude of the alternating quantity it measures.

The oscillograph tube includes an electron gun FGP (See Sect. 57) that shoots a stream of electrons against a fluorescent screen on the flattened end A of the evacuated tube illustrated in Fig. 354-1 just after emerging from the gun, the beam of electrons passes between two parallel plates DD' which may deflect the beam up or down, depending on the difference in potential applied between the plates. The beam of electrons then produces a luminous spot when it strikes the screen. If a voltage to be measured is applied between D and D', the deflection of the luminous spot will be proportional to the voltage. Hence a scale on the screen can be marked to indicate the applied voltage directly. One great advantage of the cathode ray tube as a measuring device is that there is very little time lag between the application of a given voltage and the resultant deflection of the indicating spot of light. Thus it indicates a voltage even if the voltage is changing rapidly.

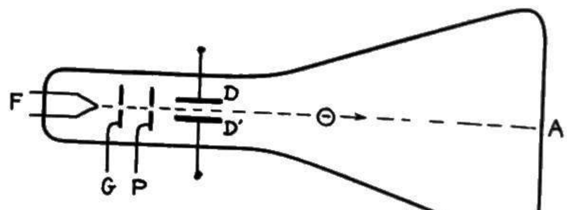


Fig. 354-1

Cathode ray tubes are usually provided with two pairs of deflecting plates placed so that the beam passes through one pair after the other and either pair can deflect the beam independently of the other. One pair of plates is placed to deflect the beam vertically while the other one is placed to deflect it horizontally.

To use a cathode ray tube to plot a graph of an alternating voltage against the time, the voltage is applied to produce a vertical deflection. At the same time, a uniformly increasing voltage is applied to the other set of plates to give a horizontal motion of the spot across the face of the tube. Thus the path of the luminous spot on the screen of the tube will be a graph of the alternating voltage plotted vertically against a horizontal time axis. The horizontal "sweep" voltage is usually obtained by slowly charging up a capacitor through a resistance. At the end of the sweep, the capacitor is suddenly discharged to return the spot to its starting position ready for the next sweep. The occurrence of the sweep cycle may be electrically synchronized with the sine wave to be observed so that each successive sweep will give a trace which is a duplicate of the preceding one, thereby permitting continuous observation.

Various types of fluorescent materials are used on cathode ray screens, depending on whether the curve is to be photographed or observed directly. For general use, it is desirable to have the visible effect of the beam persist for some little time after the beam passes over a given point. With such a screen, the first part of a curve will still be visible when the last part is finished.

A cathode ray tube as described above measures voltage. It can be used to measure a current by making the current flow through a known resistance, and measuring the potential drop across this resistance. Since this drop is proportional to the current for a given resistance, the scale on the screen can be marked to read directly in amperes if desired.

Instead of using a voltage between parallel plates to deflect the beam in a cathode ray tube, the beam can be deflected by the magnetic field of a current flowing in coils placed outside the tube. Such coils must be placed so that they produce a magnetic field perpendicular to the electron beam inside the tube. For example, a pair of coils could be placed as shown in Fig. 354-2 to give a downward deflection to a beam of electrons moving in to the paper between the two coils. The circle between the coils in this figure represents a cross-section of the narrow neck of the tube, looking toward the screen. With magnetic deflection of this type, the deflection of the spot on the screen is approximately proportional to the current in the pair of deflecting coils.

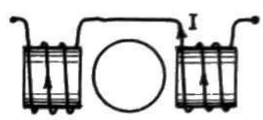


Fig. 354-2

Chapter 36

ALTERNATING CURRENTS IN CONNECTED PATHS

360. Alternating Emf in Series Paths. When two emfs are connected in series, the total instantaneous emf e applied in the combination must be the sum of the instantaneous emfs e_1 and e_2 in the two paths. This relationship follows because the principle of conservation of energy must hold at all times, including any particular instant. Thus we may write

$$e = e_1 + e_2 \ldots \tag{1}$$

If e_1, e_2 etc., are sinusoidal emfs they may be expressed in the usual form and we can write

$$e = (E_0)_1 \sin(\omega t + \alpha_1) + (E_0)_2 \sin(\omega t + \alpha_2) \tag{2}$$

The resultant e plotted against the time will thus be a curve obtained by adding two sine curves.

The most obvious method of adding two sine curves is to construct a third curve having an ordinate at any time which is the algebraic sum of the ordinates of two original curves at that time.

Two sine curves can also be added by trigonometry. We know that any sinusoidal quantity is the vertical component of a rotating vector. We also know that when we add vectors, we add their components. It follows that two sinusoidal quantities can be added by adding the two corresponding vectors, and taking the vertical component of the resultant vector. If the two vectors to be added are rotating with the same frequency, the resultant vector will have a constant length and it will rotate with the same frequency as the component vectors. The vertical component of this resultant vector will accordingly be a sinusoidal quantity. Thus we know that if we add two sine curves by adding their ordinates point by point, the resultant curve will be a sine curve. Note that the maximum ordinate of the resultant curve will be less than the sum of the maximum values for the two curves which are added, unless the two curves are in phase so that both reach their maximum values at the same time.

In case more than two curves are to be added, the same principle applies and the rotating vector for the sum is the resultant of all the rotating vectors for all the curves to be added.

361. Alternating Current in Paths Having Both Resistance and Inductance. Most actual conducting paths used in alternating currents possess both inductance and resistance. The electrical behavior of such a path can be explained by considering it to be a combination of a purely resistive path and a purely inductive path connected in series. If a pure resistance R and a pure inductance L are connected in series, the instantaneous current i flowing at any time must be the same for both R and L. Also the instantaneous emf e applied to the combination must be the sum of the instantaneous emf e_R applied in the resistive path and the instantaneous emf e_L applied in the inductive path, so that

$$e = e_R + e_L . \qquad (361\text{-}1)$$

If the current i is an alternating sinusoidal current, we may write

$$i = I_0 \sin\omega t, \qquad (361\text{-}2)$$

Then

$$e_R = I_0 R \sin\omega t \quad \text{or} \quad e_R = (E_0)_R \sin\omega t \qquad (361\text{-}3)$$

and

$$e_L = I_0 \omega L \sin(\omega t + 90^\circ) \quad \text{or} \quad e_L = (E_0)_L \sin(\omega t + 90^\circ). \qquad (361\text{-}4)$$

The total e will thus be the sum of two sinusoidal quantities which have reference vectors rotating with the same angular velocity ω. As pointed out in the preceding section, e will also be a sinusoidal quantity that may be written in the general form

$$e = E_0 \sin(\omega t + \alpha). \qquad (361\text{-}5)$$

The values of E_0 and α for this resultant emf may be found by finding the rotating vector for e. This vector A will be the resultant of the rotating vectors A_R and A_L which holds for e_R and e_L respectively. The diagram for adding these vectors is shown in Fig. 361-1 for a time when t = 0. Since the length of these vectors represent E_0, $(E_0)_R$ and $(E_0)_L$ respectively, and since the two vectors for $(E_0)_R$ and $(E)_L$ are 90° apart, it follows that

$$E_0 = \sqrt{(E_0^2)_R + (E_0^2)_L} . \qquad (361\text{-}6)$$

Also the vector for E_0 will be ahead of the vector for $(E_0)_R$ by an angle α such that

$$\tan\alpha = (E_0)_L / (E_0)_R . \qquad (361\text{-}7)$$

Thus the emf e applied to the combination will lead the current in phase by the angle α.

Fig. 361-1 shows the position of all three rotating vectors at a time t = 0. As time goes on all three vectors rotate together with the same angular velocity ω. The three corresponding sine curves for e_R, e_L, and e are shown in Fig. 361-3. Fig. 361-2 shows the rotating vectors at a time t = 1/8 period. Note that for any time such as t = (1/8)P, the ordinate for the e-curve is the sum of the ordinates for the e_R-curve and the e_L-curve at that time.

If we multiply each E_0 in Eqs. (361-6) and (361-7) by .707, we get

$$E = \sqrt{E_R^2 + E_L^2} \tag{361-8}$$

$$\tan \alpha = E_L / E_R \tag{361-9}$$

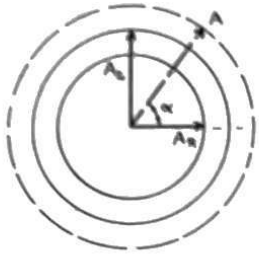


Fig. 361-1

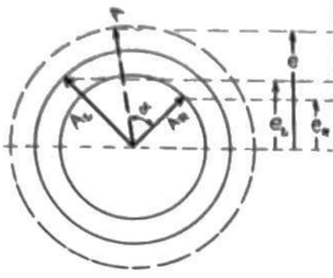


Fig. 361-2

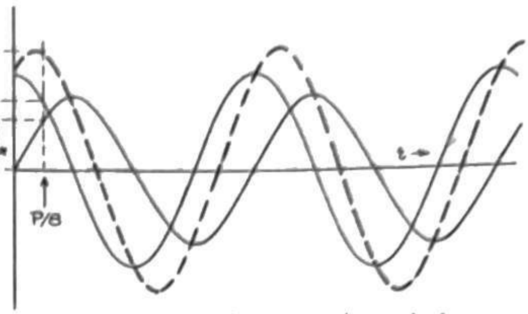


Fig. 361-3

362. The Vector Addition of Impedances in Series. The emf e required to send a current $i = I_0 \sin\omega t$ through a number of impedances in series will in general have some magnitude E_0 and some phase angle $(\omega t + \alpha)$. The values of E_0 and α will depend on the combination of impedances. An impedance with a magnitude E_0/I_0 and an angle α is called the equivalent impedance of the combination.

The equivalent impedance of several impedances in series can be found by adding the separate impedances as vectors. To show why this is true consider an inductance L and a resistance R in series. The effective emfs across L and R and the emf across the combination are related as shown in Fig. 362-1 for a time when the phase angle of the current is zero. The vectors representing the emfs will then coincide with the vectors for the impedances according to Sect. 349. The respective impedances will thus be as shown in Fig. 362-2. This shows that the impedance Z of the series combination is the resultant of the vectors R and ωL, just as E is the resultant of E_R and E_L in Fig. 362-1. To illustrate with numerical values, let us suppose that I = 10 amp while E_R = 30 v, E_L = 40 v and E = 50 v. Then R = 3Ω, ωL = 4Ω, Z = 5Ω and the angle ϕ is equal to the angle α. The magnitude of Z is thus given by

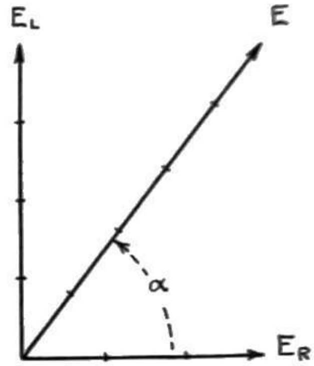


Fig. 362-1

$$Z = \sqrt{R^2 + (\omega L)^2} \tag{362-1}$$

and the angle ϕ of the impedance is given by the equation

$$\tan\phi = \omega L / R. \tag{362-2}$$

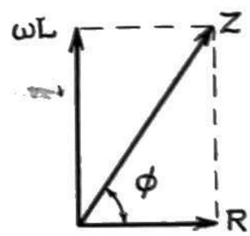


Fig. 362-2

363. Series Combination of R, L and C. In any series combination of paths, the current is the same for all the paths and the instantaneous value e of the voltage across the combination is the sum of the instantaneous voltages across the separate paths. Thus the rotating vector for e will be the resultant of the rotating vectors E_L, E_R and E_C that apply to the inductance, resistance and capacitance respectively. Fig. 363 is a vector diagram showing the addition of these vectors to give a resultant E. This diagram is drawn for a time when the phase angle of the current is zero, so that all angles shown will be relative to the angle of the common current.

The addition of the three vectors E_L, E_R and E_C in Fig. 363 has been carried out by first

adding E_L and E_C, and then combining that resultant as a single vector with E_R to get the resultant of all three. This is done because E_L and E_C are opposite to each other, and hence can be combined by simple algebraic addition. It follows from the diagram that the magnitude of E will be given by

$$E = \sqrt{E_R^2 + (E_L - E_C)^2}$$

It is to be noted on the diagram that the voltage across the whole combination may be less than that across the inductance alone, or less than that across the capacitance alone. It cannot, however, be less than that across the resistance alone.

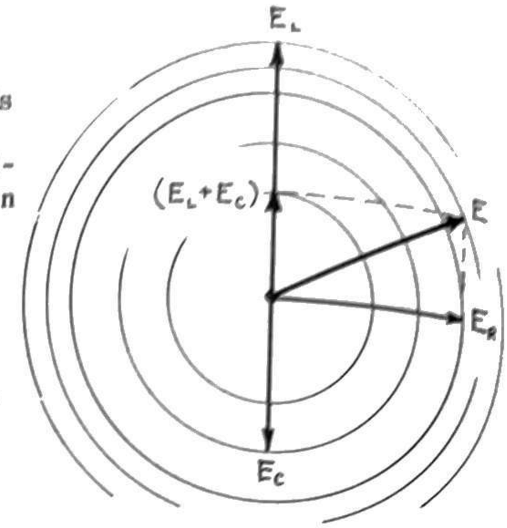


Fig. 363

364. Impedance of a Series Combination of R, L and C. If a resistance R, an inductance L, and a capacitance C, are connected in series, the resultant impedance of the combination may be found by adding the vectors of the separate impedances as explained in Sect. 362. A vector diagram showing this addition is given in Fig. 364. Since ωL and $1/\omega C$ are directly opposite, they have been added first by simple algebraic addition to give a single upward vector having a magnitude $(\omega L - 1/\omega C)$. This vector is then combined with the vector R to give the resultant Z of all three original vectors. It follows from the figure that

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \qquad (364\text{-}1)$$

and the angle ϕ of the resultant impedance will be such that

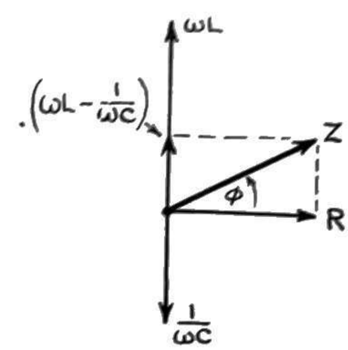


Fig. 364

$$\tan\phi = (\omega L - 1/\omega C)/R \qquad (364\text{-}2)$$

365. Resonant Circuits. If a given emf E be applied to an inductance L, a resistance R, and a capacitance C in series as shown in Fig. 365-1, the current I will be given by Eq. (364-1) as

$$I = \frac{E}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \,. \qquad (365\text{-}1)$$

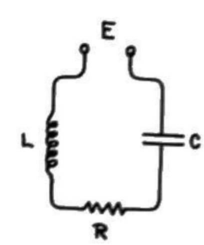


Fig. 365-1

If the frequency be varied with a constant emf, there will be one particular frequency for which the term $(\omega L - 1/\omega C)$ is zero. At that frequency, the denominator of Eq. (365-1) will be less than for any other frequency, either larger or smaller. Hence the current will be larger for that frequency than for any other. This critical behavior of a series combination of L,C, and R is illustrated in Fig. 365-2 where the current I is plotted against the angular frequency ω for a constant applied emf.

Circuits in which the ratio of current to voltage has a maximum value at some particular frequency as illustrated in Fig. 365-2 are called "resonant" circuits. The frequency at which the current is a maximum is called the resonant frequency ω_0. Since resonance occurs when

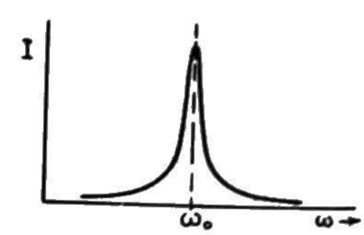


Fig. 365-2

$$(\omega L - 1/\omega C) = 0 \qquad (365\text{-}1)$$

it follows that

$$\omega_0 L = \frac{1}{\omega_0 C} \tag{365-2}$$

and

$$\omega_0 = 1/\sqrt{LC} \tag{365-3}$$

The significance of resonance as seen on the impedance diagram of Fig. 364 is that the two vertical vectors ωL and $1/\omega C$ will be equal and opposite and thereby cancel each other. Hence the total impedance of the combination at resonance has a minimum value simply equal to the resistance R.

The peculiar property of resonant circuits as illustrated in Fig. 365-2 makes them suitable for use as selective "tuning" circuits in a radio receiver. In such a receiver, emfs of different frequencies from different broadcasting stations are all applied in the same series circuit. By means of a variable condenser, the resonant frequency of the circuit can be adjusted so that the current produced by the emf from one particular station will be relatively large while the currents from the other emfs are negligible.

Chapter 37

POWER IN A.C. PATHS

370. Power Expended in an A.C. Path. Power Factor. The power expended in an a.c. path at any instant will be the value of the emf e at that instant multiplied by the value of the current i at the same instant. Since both e and i fluctuate in value, the product ei will also fluctuate in value from one part of a cycle to the next.

To write an expression for the average expenditure of power over any whole number of cycles, let us consider any interval of time starting when the phase angle of the current is zero. The values of i and e may then be written

$$i = I_0 \sin \omega t \tag{370-1}$$

and

$$e = E_0 \sin (\omega t + \phi). \tag{370-2}$$

Here ϕ is the angle by which the emf leads the current in the path under consideration. In other words, ϕ is the angle of the impedance of the path. From these expressions it can be shown by trigonometry (See Sect. 371) that the average value P of the product ei is given by

$$P = EI \cos \phi. \tag{370}$$

Here E and I are the effective values of the voltage and current respectively. The quantity $\cos\phi$ is called the power factor of the path under consideration.

For a purely resistive path, the angle ϕ is zero, and the power factor $\cos\phi$ is unity. The power expended in a resistive path is thus EI, or I^2R. For any other type of path, whether the voltage leads or lags the current, the power factor will have a value less than unity. For either a pure inductance or a pure capacitance, the value of the power factor is zero. This means that no power can be expended in such a path regardless of how much current flows in the path.

An examination of the curves of e and i for an inductive path as given in Fig. 347 will show why the average expenditure of energy is zero in any path where ϕ is 90°. Note that during the first half of the surge of current between the points A and B, the emf is in the same direction as the current and the emf expends energy. During the second half of the same surge of current, (that is from B to C on the figure), the emf is reversed and the current does work on the emf

until the current is stopped by the emf. Thus the emf alternately expends energy in building up a current in a given direction, and then receives it back again when it reverses and stops the current. The net expenditure of power is therefore zero.

It follows from above that an inductance or a capacitance can be inserted in series with a given load to control the magnitude of the current without introducing any waste of energy. For example, a coil in which the inductance can be varied by inserting an iron core is sometimes used in series with theater lights so they can be dimmed at a slow, uniform rate.

In transmitting power it is desirable to have the power factor as large as possible. A smaller power factor requires a larger current to transmit a given amount of power at a given voltage, and thus causes more heat loss in the resistance of the transmission line, and in the internal resistance of the generator.

371. Derivation of Expression for Average Power. To derive the expression for the average power given in Eq. (370) above, let us write the expression for the instantaneous power in the form

$$p = ei = E_0 \sin(\omega t + \phi) \times I_0 \sin\omega t. \tag{371-1}$$

Here ϕ is the angle of the impedance to which the equation applies. Expanding this product by trigonometry gives

$$p = E_0 I_0 (\sin^2\omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi). \tag{371-2}$$

The average value of $\sin^2\omega t$ over any whole number of periods is 1/2. (See Sect. 345). Since $\sin\omega t \cos\omega t = (\sin 2\omega t)/2$, the average value of the last term in Eq. (371-2) is zero. Thus the average value P of the instantaneous power p taken over any whole number of periods will be given by

$$P = (E_0 I_0/2) \cos\phi \tag{371-3}$$

or

$$P = EI \cos\phi \tag{371-4}$$

Chapter 38

TRANSFORMERS

380. A transformer is a device to convert the voltage of an alternating power supply to either a higher or lower voltage. By the use of transformers, the power from an ordinary 110 volt outlet can be stepped up to operate a 100,000 volt x-ray tube or stepped down to operate a 10 volt toy train. The possibility of changing the available power in this way by relatively simple transformers is one of the great advantages of a.c. power over d.c. power.

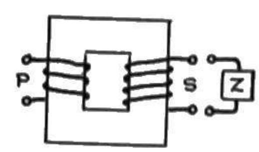


Fig. 380

The essential parts of a transformer are a "primary" coil P which is connected to the source of power, a secondary coil S in which the output voltage is induced, and a magnetic core threading through both coils. The core is usually an endless ring of iron as illustrated. The two coils are electrically insulated from the core and from each other. The general principle of operation of a transformer is that an alternating emf applied to the primary coil produces an alternating current in that coil and thus an alternating magnetic flux in the core; this alternating flux induces an emf in the secondary coil which may be of different magnitude than that applied to the primary coil. The induced voltage which the secondary coil furnishes to the final "load" Z will be larger or smaller than the original applied voltage depending on whether the secondary coil has more or less turns than the primary coil.

In order that the primary coil may accept energy from the power line, it must exert a back emf against which the applied emf can do work, and this back emf must be equal and opposite to the applied emf. Since the back emf is furnished by the self inductance of the primary coil, the magnitude of the instantaneous applied emf e_1 will be given by

$$e_1 = n_1 \, (d\Phi/dt) \tag{380-1}$$

where n_1 is the number of turns in the parimary and Φ is the magnetic flux established in the core by the primary current. The emf e_2 induced in the secondary will be given by

$$e_2 = n_2 \, (d\Phi/dt) \tag{380-2}$$

where n_2 is the number of turns in the secondary. Since the same endless core passes through both the primary and secondary coils, $(d\Phi/dt)$ is the same for both and hence

$$\frac{e_2}{e_1} = \frac{n_2}{n_1} \tag{380-3}$$

To secure efficient operation as well as other desirable characteristics, an "ideal" transformer should have primary and secondary coils with negligible resistance, and should have endless core of infinitely high permeability with no magnetic hysteresis. With such a transformer, all the energy applied to the primary coil will be given out without loss by the secondary coil. Good commercial transformers approximate these specifications so closely that they may have efficiencies as high as 99%.

A sinusoidal emf applied to an ideal transformer will give a sinusoidal emf of the same frequency in the secondary, and the ratio of the effective a.c. values of the emfs will be the same as the ratio of the instantaneous values given in Eq. (380-3) above.

An ideal transformer is the electrical counterpart of an ideal simple machine in mechanics, with the emf corresponding to the force and the current corresponding to the velocity. It will be recalled that in an ideal machine, the power passes through from input end to output end without any change in amount, but it may come out as a different combination of force and velocity. Thus if the force is greater at the output end, the velocity must be less, and vice versa. Similarly the power which is applied to the primary side of an ideal transformer comes out at the secondary side without any change in amount, but it may come out as a different combination of emf and current. It follows that if the output voltage on the secondary side is greater, the output current will be less, and vice versa. For an ideal transformer this relationship between the effective values for the currents and voltages may be written

$$E_2 I_2 = E_1 I_1 \quad \text{or} \quad I_2/I_1 = E_1/E_2 \tag{380-4}$$

Transformers are used in connection with lines for transmitting power over long distances. The resistive power loss in such transmission is equal to $I^2 R$ where R is the resistance of the line. Hence if the power to be transmitted is transformed so that it has a high voltage and small current, the same amount of power can be transmitted through the resistance of the line with less loss. At the receiving end of the line the power can be transformed again down to a lower and safer voltage for use in motors, lights, etc.

Note that when the secondary circuit is not completed through a load, the transformer cannot pass any energy out at the secondary side. In that case, an ideal transformer will not accept any energy from the primary side and no primary current can flow. When a connection to the secondary circuit is made, enough primary current will flow to furnish the energy being expended by the secondary coil. Actual transformers may draw a small current on the primary side even when the secondary side is not furnishing a current. With doorbell transformers, this current is so small that the primary of the coil is often permanently connected to the power line, with the switch in the secondary.

Chapter 39

ELECTRIC OSCILLATIONS

390. If some of the positive charges in a conductor are displaced toward one end by a temporary disturbance, there will be an excess of positive charge crowded at one end while an equal excess of negative charge is left at the other end. The resulting potential difference will then force the displaced body of charge back toward its equilibrium position. The motion of the charge back toward its equilibrium position will constitute a current I flowing from the high potential end toward the low potential end as indicated in Fig. 390-1. The rate of flow will increase as long as any potential difference remains, and hence the current will continue to increase until the body of charge has returned to its equilibrium position. On account of inductance, this current cannot stop unless there is a potential difference in the opposite direction to stop it. Thus the charge will continue to move past its equilibrium position until the positive charge accumulating at the opposite end brings the motion to a stop. The situation is then just like it was at the start except that the positive charge is now displaced in the opposite direction. This same sequence of events will repeat itself over and over, first in one direction, and then in the other. Such a surging of charge back and forth in a conductor is called an electrical oscillation, and it is similar in many respects to the vibration of a mass under the action of an elastic restoring force.

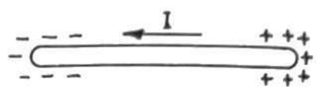


Fig. 390-1

Electric oscillations may also occur in a circuit consisting of a capacitance and an inductance as shown in Fig. 390-2. Here the inductor consists of a single turn of wire connecting the two plates of the condenser. Charges may oscillate around through this inductance from one plate of the condenser to the other, just as they may surge back and forth through a straight conductor.

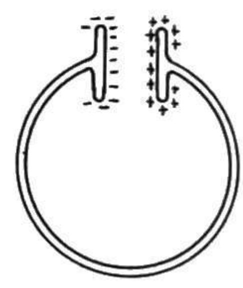


Fig. 390-2

A similar circuit is shown in Fig. 390-3 where an inductor having many turns is connected from the one plate of a condenser to the other. The displacement of the charge which is needed to start an oscillation in such a circuit may be obtained by charging the condenser before the inductance is connected to it. After the inductance is connected, the condenser will discharge through the inductance L with a current I in the direction shown by the arrow tip. By the time the condenser is completely discharged, the current will have acquired enough energy to keep it flowing by its own power. It will continue to flow until its energy is expended in charging the condenser up in the opposite direction. The sequence of events will then be repeated in the opposite direction, and as time goes on, the charges will surge back and forth from one plate to the other with an oscillatory motion through the inductance.

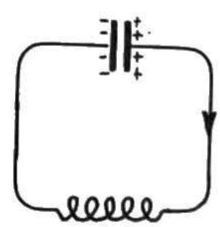


Fig. 390-3

The oscillation of charges in a circuit as described in the preceding paragraph constitutes an alternating current flowing around in the closed circuit. A detailed mathematical analysis shows that this current will be a sinusoidal function of the time. An oscillation in such a circuit may therefore be considered as an ordinary sinusoidal alternating current which oscillates of its own accord once it gets started.

A circuit consisting of a capacitance and an inductance connected in series is often referred to as a resonant circuit because it will show the phenomena of resonance as described in Sect. 365.

391. Detection of Electric Oscillations. Electric oscillations in a circuit may be detected by a cathode ray oscillograph connected as a voltmeter across the condenser. As the charges oscillate, the potential difference between the plates will change back and forth from a maximum in one direction to a maximum in the other direction. An oscillograph may therefore be connected to the condenser so that the alternating voltage will deflect the spot up and down on the screen, giving a visual indication of the oscillations.

Oscillations in a circuit may also be detected by inserting an a.c. ammeter in the circuit to measure the alternating flow of charge.

392. Frequency of Electric Oscillations. The frequency of an electrical oscillation depends on the circuit in which it occurs. The frequency of a circuit consisting of an inductance L connected to a capacitance C depends upon the values of the inductance and capacitance according to the equation

$$f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \tag{1}$$

Thus the frequency will be smaller if either the inductance or capacitance is made larger.

A wide range of frequencies may be had in practice with electrical oscillations. Low frequency oscillations having a period of several seconds can be obtained using a bank of condensers and inductors which can be accommodated on a small table top. Frequencies as high as 2,000,000,000 cycles per second can be had by reducing the capacitance and inductance to a point such that the circuit consists of one open loop of wire about the size of a finger ring.

The frequency of oscillation in a straight rod is determined by the same general factors which govern the frequency in a circuit. For an oscillation in a straight rod, the two ends correspond to the two plates of the condenser in a resonant circuit. The center part of the rod between the ends corresponds to the inductor of a resonant circuit. This capacitance and this inductance will both be very small. The frequency of an oscillation in a short rod will accordingly be very high and it will be inversely proportional to the length of the rod. Oscillations in a rod one inch long will have a frequency of about 6,000,000,000 cycles per second.

393. The Energy of Electric Oscillations. An electric oscillation has a certain amount of energy associated with it. For example, in the oscillatory discharge of a condenser, the energy which was stored in the charged condenser becomes the energy of the oscillation. As the condenser discharges, it expends its energy to build up the discharging current. This energy will then be expended to charge the condenser up again in the opposite direction, and so forth. As the charges oscillate, the energy will accordingly be passed back and forth from one form to the other, much as the energy of a swinging pendulum is transformed back and forth between the potential and kinetic forms.

The total energy of an oscillatory discharge would not decrease if there were no losses of energy such as resistive losses in the circuit. In a lossless circuit, the voltage of the condenser would be the same in magnitude each time it was charged to a maximum value in either direction. The oscillation would therefore persist indefinitely without any decrease in amplitude. In any actual circuit there will always be some loss of energy in each surge of current, so that the amplitude of the oscillation will gradually die out. For example, if a condenser is being discharged through a coil of wire, there must be some resistance along with the inductance, and the voltage of the condenser will alternate back and forth with decreasing amplitude. A graph of the voltage plotted against time would accordingly appear as shown in Fig. 393. The greater the resistance of the circuit, the more rapidly the oscillations will die out. If the resistance exceeds

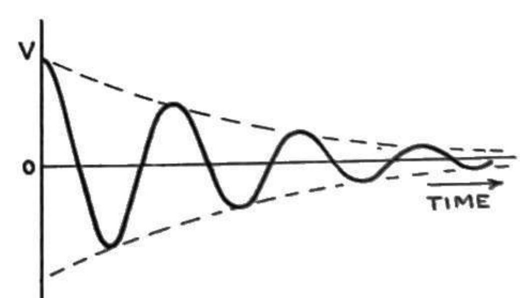


Fig. 393

a certain value, no oscillation will occur because all the energy will be expended in the first surge of current.

394. Oscillations and Resonance. The amplitude of an electric oscillation in a resonant circuit can be maintained by supplying energy from some external source. This energy can be furnished by an alternating emf if the emf has the same frequency as the oscillation. The alternating force of such an emf will then change its direction just as often as the motion of the oscillating charges is reversed, and the emf can be applied so that its force will always act in the direction of the motion.

An alternating emf may be applied in a resonant circuit by placing the inductance near another coil in which an alternating current is furnished by a generator. The inductance of the resonant circuit will then have an emf induced in it in the same way that an emf is induced in the secondary of a transformer. Two circuits arranged in this way so that energy is transferred from the one to the other are referred to as coupled circuits.

An emf may also be applied in a resonant circuit by opening the circuit and inserting an a.c. generator in series. This arrangement has already been discussed in Sect. 365, and is represented in Fig. 365-1. Note that the relatively large alternating current which flows at resonance as described there may be considered as an electrical oscillation in which the energy losses are neutralized by an applied emf of the same frequency. This unified point of view is emphasized by the fact that the frequency at which resonance occurs with an applied emf is the same as the frequency with which the charges would oscillate by themselves if there were no resistance in the circuit. The presence of resistance in a resonant circuit reduces the frequency of free oscillations somewhat, but it does not change the resonant frequency.

An electric oscillation in a straight conductor may be maintained by connecting it to an a.c. circuit as shown in Fig. 394. There a section of the central part of the rod is used to complete a circuit in which an oscillator is generating an a.c. emf. Some of the energy in the closed circuit will then be transferred to maintain the oscillation of charges from one end of the rod to the other.

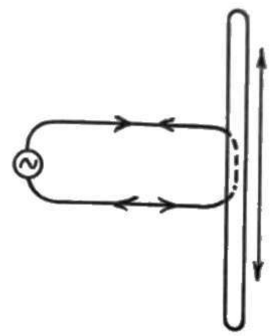


Fig. 394

Chapter 40

ELECTROMAGNETIC WAVES

400. If an electric charge Q is displaced a certain amount, the field of this charge as measured at some distant point P will change accordingly. However, the effect of the displacement will not be observed at the distant point immediately. The lapse of time between the displacement of Q and the observed effect at P is found to be proportional to the distance from the charge to the point where the field is being measured. This indicates that the effect of the disturbance travels out from the charge with a definite velocity. This velocity can be measured by experiment and is found to be the same as the velocity of light.

It follows from above, that whenever an electric charge oscillates, there will be a corresponding fluctuation in the field of this charge in the surrounding space. These fluctuating disturbances will move out from the oscillating charge with the velocity of light, and thereby constitute what is known as an electromagnetic wave.

To illustrate the nature of an electromagnetic wave, let us consider the electric field due to a charge which is oscillating up and down in a straight conductor. For example, Fig. 400 shows

04 - 63 X

a vertical rod AB which might be the antenna of a broadcasting station. Whenever a positive charge is at the top end of the rod, an equal negative charge will be at the bottom end, as shown, and vice versa. As the charge oscillates, the result is as if the positive and negative charges exchanged places every half cycle. If the charges on the rod AB are at rest in the position shown, then the field E at a point P will be a downward field which is the resultant of a repulsive field E_1 and an attractive field E_2. If the positions of the charges are reversed, E will be directed upward. Hence as the charges oscillate, the direction of the field at P will alternate, but it will always be vertical. From the point of view of an observer at P, an electromagnetic wave therefore would appear as an alternating, vertical electric field. At another point P' somewhat further away from AB, the wave will also appear as an alternating, vertical electric field, but the alternations will be behind those at P in phase by an amount that depends on the time required for the wave to travel from P to P'.

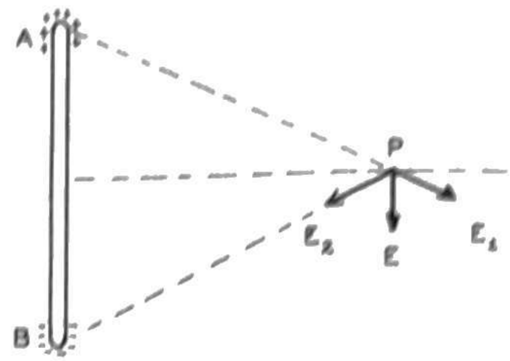


Fig. 400

As stated above, the oscillations of the charges on AB creates an alternating wave that travels away from AB with the velocity v of light. If T is the period and f the frequency of oscillation of the charges, the wave will travel away from AB a distance λ in one period where

$$\lambda = vT = v/f. \tag{400}$$

If P_1 is a point a distance λ away from AB, the field at P_1 at any given time t depends on position of the charges on AB one period before. That is the same as it is at the given time t. At a point P_2 a distance 2λ away, the field depends on the position of the charges two periods before, and that also is the same as it is at the given time t. The same will be true for points at distances 3λ, 4λ and so on. This distance λ from any one point to the next point where the alternating field has the same phase, is called the wave-length of the wave.

The wave length corresponding to any frequency may be computed by Eq. (400) using the known value of v as 3×10^8 meters/sec. Thus a frequency of 1000 kilocycles, which lies in the commercial broadcast range will have a wave length of 300 meters. A frequency of 10,000 megacycles such as has been used in radar gives a wave length of 3 cm. The common 60 cycle per second frequency of commercial power lines gives a wave length of 5×10^6 meters or 3,100 miles.

A charge oscillating in a straight conductor sets up an alternating magnetic field in the surrounding region along with the alternating electric field described above. When the positive charge is moving down, the current will produce magnetic lines of force which are horizontal circles centered at the conductor. A half cycle later when the positive charge is moving up, the horizontal circles will be directed around in the opposite direction. At a point such as P in Fig. 400 therefore, there will be a horizontal alternating magnetic field perpendicular to the plane of the drawing. This alternating magnetic field also spreads out with the velocity of light along with the alternating electric field. The term electromagnetic wave applies to the combined effect of both fields.

The above description of electromagnetic waves was based on our knowledge of steady electric and magnetic fields. Additional factors must be taken into account in a complete analysis, but the nature of the resultant wave from an oscillating charge is essentially as described above. In general, an electromagnetic wave at any point in free space relatively far from the source may be observed as an alternating electric field and an alternating magnetic field. Also under these conditions, the two fields will be perpendicular to each other and both will lie in a plane which is perpendicular to the direction in which the wave is traveling away from the source.

401. The Reception of Electromagnetic Waves. The alternating field of an electromagnetic wave will constitute an alternating emf acting in the region where the field exists. This emf may produce an alternating current in a suitable conductor used as a receiving antenna, and this current may be measured by some device such as an ammeter. By letting the receiving antenna be a conductor or a circuit which can oscillate with the frequency of the incoming wave, a large resonant current can be obtained as explained in Sec. 365.

The resonant effect also makes it possible to select some one wave from which energy is to be received by an antenna when several waves of different frequencies are striking it at the same time. This selective effect can be demonstrated by using a vertical antenna as illustrated in Fig. 400 of this chapter. Two wires connected to a lamp bulb as shown in Fig. 401 may be used as a receiving antenna. Such a receiving antenna constitutes a long conductor in which charges may oscillate from one end to the other. As the charges pass through the center of the antenna, they must pass the lamp bulb and the current can be detected by the light produced. An a.c. ammeter can be used instead of the lamp to observe the current.

Fig. 401

402. Energy of Electromagnetic Radiation. Energy is radiated away from an oscillating charge by the electromagnetic wave which moves away from the charge. Thus any oscillation would lose energy in the form of radiation even if there was no resistance in the conductor where the oscillation occurs. This makes any circuit appear to have an added resistance for alternating currents which does not exist for direct currents. In general, radiation losses are relatively unimportant unless the geometrical dimensions of a circuit are of the same order of magnitude as the wave length of the radiation.

The lighting of an incandescent lamp by the current induced in a receiving antenna furnishes an example of energy being transported by an electromagnetic wave.

Chapter 41

THERMIONIC TUBES

410. Thermionic Emission. When a piece of metal is heated to a relatively high temperature, some of the electrons in the material may acquire enough thermal energy to escape through the surface. The escape of these electrons is similar to the evaporation of molecules from the surface of a heated liquid. After the electrons escape from the metal, they will behave as free charged particles restrained only by their own inertia. The emission of electrons from a body of material because of its temperature is called thermionic emission. The process of thermionic emission is of great practical interest since it is an essential feature in the operation of most of the tubes used in electronic devices today.

The simplest thermionic tubes are the highly evacuated, two element tubes which are used in power supplies to furnish d.c. voltage from an a.c. source. These so-called "diodes" consist of a heated "cathode" K which emits electrons and a plate P which may be maintained at a positive potential to collect the electrons. Both of these elements are inclosed in an evacuated glass or metal tube, with connections extending out through the wall to external terminals. In some tubes, the cathode is heated indirectly by a filament F of fine wire connected to a battery A or to a small transformer. In other tubes, the filament itself serves as the heated cathode.

The emission of electrons from a cathode can be demonstrated by applying a difference in potential V between the plate and cathode by means of a battery B as shown in Fig. 410-1.

Electrons emitted from K will then move over to P. As far as events outside the tube are concerned, this is equivalent to a positive current flowing from P to K. The path through the tube therefore completes a closed circuit PKBP, in which a current I may flow as indicated by the current meter M.

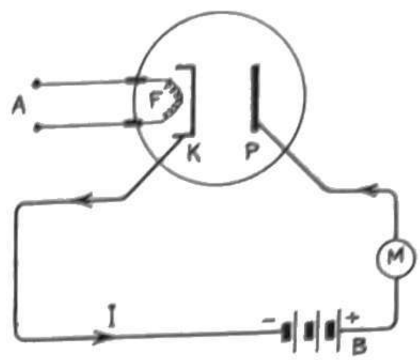


Fig. 410-1

The maximum current which a thermionic tube can conduct is limited by the number of electrons emitted from the cathode per unit of time, and that in turn depends only on the nature and the temperature of the cathode. The relationship between the maximum current I_m and the absolute temperature T of the cathode is given by the equation

$$I_m = AT^2 e^{-b/T} \qquad (1)$$

where A and b are constants depending on the cathode. The derivation of this equation may be found in more advanced books and it can be verified by experiment.

Tungsten metal is sometimes used as a cathode material because it can be heated to a high temperature and may be subjected to high voltages without any deterioration of its emitting surface. The maximum thermionic current per square centimeter that can be obtained from tungsten at various temperatures is shown by the curve T in Fig. 410-2. It is to be noted that appreciable emission does not begin until the tungsten reaches a white hot temperature at which most of the common metals would be melted. Relatively large emission currents can be obtained at much lower temperatures by coating the emitting surface with a thin layer of certain oxides. Other metals than tungsten can thus be used as cathodes at a dull red temperature which will not melt the metal.

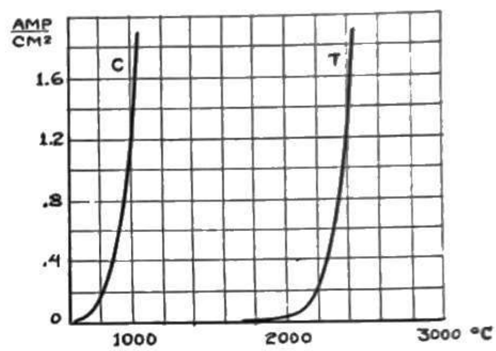


Fig. 410-2

411. Space Charge. At any instant when a current is flowing through a thermionic tube, there must be a considerable cloud of electrons between the cathode and the plate. Because of the attraction of the positive plate, the electrons will be accelerated toward the plate and they will move faster as they get closer to the plate. For that reason, the cloud will be less dense near the plate as illustrated in Fig. 411. Such a cloud of electrons is often referred to as a space charge, and this space charge will exert electrostatic forces just like any other body of negative charge. If there is a considerable body of negative space charge, it will tend to neutralize the attraction of the positive plate and it may repel some of the newly emitted electrons back into the cathode. Thus a cloud of space charge may limit the current flowing through the tube to a value less than the maximum current that could flow at the existing temperature of the cathode. Under this condition, the amount of current will be determined by the amount of space charge.

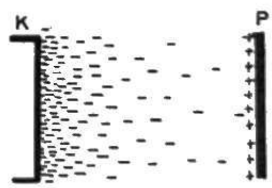


Fig. 411

The density of the space charge between the cathode and the plate at any time will depend on how rapidly the electrons are moving and therefore upon the voltage which is applied to pull the electrons toward the plate. At the lower voltages, a more dense

cloud of space charge will accumulate and more of the emitted electrons will be repelled back into the cathode. The maximum current for a given cathode temperature can thus be obtained only if the voltage is high enough to pull the electrons away from the cathode as fast as they are emitted.

In most applications, thermionic tubes are used with relatively low voltages so that the current depends upon the difference in potential applied as explained above. For a diode, this relationship between the current I and the difference in potential V between the plate and cathode may be expressed by the equation

$$I = k V^{3/2} \tag{411}$$

where k is a constant depending on the dimensions of the tube. A current which is limited by the voltage so that it obeys this equation cannot be increased by increasing the temperature of the cathode because electrons are already being emitted faster than they are being taken away.

412. The Characteristics of a Diode as a Conductor. As stated above, a two element thermionic tube may serve as a conducting path forming part of a circuit. To describe the electrical characteristics of the path through a diode, it is necessary to know the value of the current that will flow for any given difference in potential between the ends of the path. This may be done by specifying the value of k for a given tube in Eq. (411) above, or it can be done by plotting a graph of I vs V. Such a graph is shown in Fig. 412 for a commonly used commercial tube. Within the limits shown on the figure, the voltage never reaches the value that would be required to eliminate the effect of the space charge.

Although the electrical path through a diode does not obey Ohm's law, it is like a simple resistive path in that there will be a drop in potential along the path whenever a current flows. For any value of the current, the path may be said to have a resistance which is the ratio of V to I, but this ratio will be different for different values of the current. Such a path may be connected in series or parallel with other conducting paths, and the usual rules for currents and voltages in series and parallel paths will apply.

It is of interest to note that the electrical energy expended in the potential drop through a tube will be dissipated in the form of heat as is the case in a metallic conductor. The energy expended along the path is temporarily stored in the increasing kinetic energy of the electrons, and then it is converted into heat when the electrons strike the plate. In tubes which handle a considerable amount of power, special provision must be made to keep the plate from getting too hot.

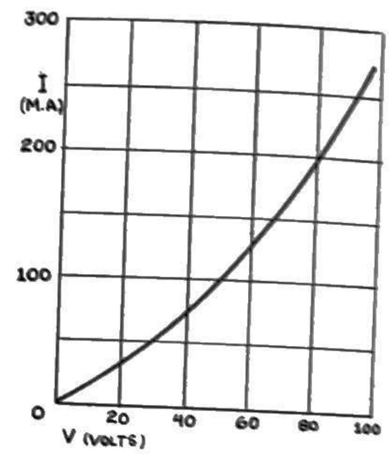


Fig. 4

Fig. 412

One very important characteristic of the path through a thermionic tube is that current can flow in only one direction through the path. For a voltage that tends to drive electrons from the plate to the cathode, no thermionic current can flow because no thermionic electrons can escape from the cold plate. In this respect, a diode serves as an electrical check valve, which allows current to flow in one direction but not in the other.

413. The Use of Diodes as Rectifiers. Diodes are commonly used to obtain direct current from an alternating current source of power. One arrangement for doing this is shown in Fig. 413-1 where the alternating power comes from the secondary MN of a transformer. The resistance AB represents the path through which a direct current is desired. During the positive half of each cycle, the emfs e_1 and e_2 induced in the two halves of the secondary are directed from M to C and from C to N. A current I_1 will accordingly flow around in the top loop of the

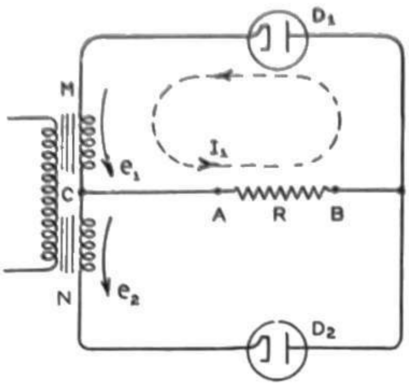


Fig. 413-1

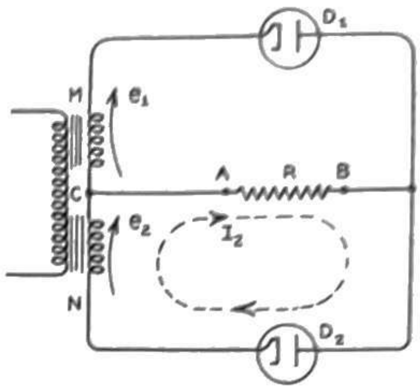


Fig. 413-2

network as indicated in Fig. 413-1. No current will flow in the lower loop because the diode D_2 will not conduct a current in the direction in which the emf e_2 is acting. A half cycle later, the direction of e_1 and e_2 will be reversed as indicated in Fig. 413-2. The diode D_2 will then allow a current to flow in the lower loop as shown, but D_1 will not conduct in the upper loop. In either case, current flows in the same direction through the middle branch of the network from A to B, with A being at a higher potential than B.

The rectified current flowing through AB will be a pulsating current whose magnitude changes with the time as shown in Fig. 413-3. Note that this curve is obtained from a sine curve by merely inverting all the negative peaks. These pulsations can be filtered out by an electric filter. A simple filter may be had by connecting an inductance in series with the resistance R and then connecting a condenser in parallel with the two as indicated in Fig. 413-4. The inductance offers a high impedance to any variation in the current through R, but allows a steady current to flow. The capacitance offers a low impedance by-pass to carry alternating current around the resistance without affecting the flow of direct current through R. Additional stages of filtering can be had by alternately adding inductances in series and capacitances in parallel to the combination of Fig. 413-4.

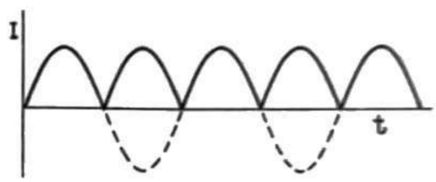


Fig. 413-3

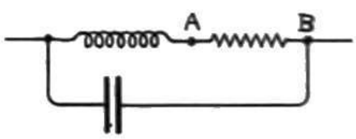


Fig. 413-4

Step-up transformers may be used with diode rectifiers to give d.c. potentials which are many times higher than the a.c. voltage which is used as a source of power in the primary of the transformer.

414. Triodes. A triode is a thermionic tube with an open mesh grid G placed across the conducting path between the emitting cathode and the plate P as illustrated in Fig. 414-1. Since the grid, the cathode and the plate are each independently connected to separate terminals outside the tube, such a tube is known as a three-electrode

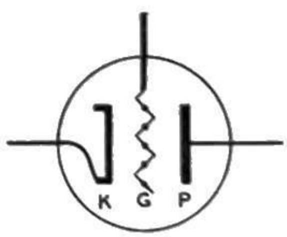


Fig. 414-1

tube, or triode. The grids of triodes usually resemble miniature wire fences in that they consist of fine wires fastened together in a two-dimensional array with relatively large open spaces left between the wires.

The function of the grid in a triode is to control the flow of electrons from the cathode to the plate. Since these electrons must flow through the openings in the grid to reach the plate, the number which get through may be reduced by charging the grid negatively from the outside. A negative charge on the grid structure will tend to repel the emitted electrons back toward the cathode and the amount of current can thus be reduced by any desired amount or cut off completely by a proper amount of negative charge applied to the grid. Since a triode constitutes a conducting path with an electric gate which can be either partly or completely closed, it may be regarded as an electric "valve." In fact, the use of triodes in electrical circuits corresponds closely to the use of mechanical valves in pipes to control the flow of fluids. A triode is like a diode in that it can conduct current in only one direction.

Triodes are generally used by making them a part of an electric circuit as shown schematically in Fig. 414-2. There the tube, the resistance R and the battery B constitute a simple series circuit PKLMNOP. The resistance R may be any resistance such as an incandescent lamp in which we wish to expend energy available from the battery B. This resistance in which the energy is to be expended is referred to as the "load," and the function of the triode in the circuit is to control the rate with which energy is expended in the load by controlling the current through the load. The current which flows in the circuit PKLMNOP is commonly referred to as the plate current because it is the current which flows into the plate terminal of the triode. Similarly the battery B in this circuit is referred to as the plate battery because it is the battery which maintains the plate at a positive potential relative to the cathode.

Triodes are generally used with the grid at a negative potential relative to the cathode. This negative potential can be had by connecting a battery C between the grid and cathode as shown in Fig. 414-2. The grid extends into the stream of electrons which constitutes the plate current through the tube, but it remains effectively insulated from it. This is because the negative grid repels the electrons, making them pass through the openings in the grid without striking the grid itself. It follows that no appreciable current flows to or from the grid through the grid battery C, and hence very little power is needed to change the grid potential. In other words, a small amount of power applied to the grid can control the expenditure of a relatively large amount of power by the plate battery in the plate circuit.

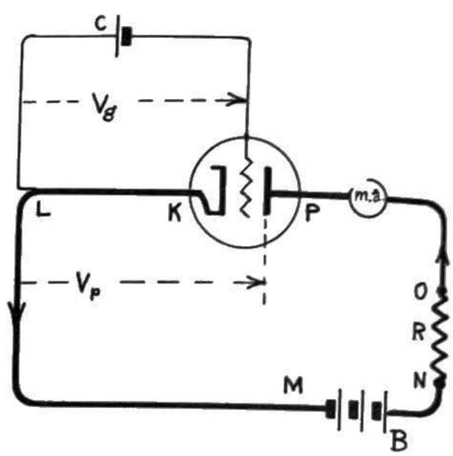


Fig. 414-2

415. <u>The Use of Triodes as Amplifiers.</u> If a triode is connected as shown in Fig. 414-2 to control the expenditure of energy from the battery B in the load R, it can be arranged so that a small change in the potential applied to the grid will produce a much larger change in the output potential across the load. In that case the tube is said to act as a voltage amplifier, and such amplification will be considered in detail in Sect. 425. As previously noted, it is also true that a small amount of power applied to the grid can control the expenditure of a relatively large amount of power in the load resistance R. When this occurs, the tube is said to act as a power amplifier.

To illustrate the use of a triode as a power amplifier, let us consider how a triode may be used to emplify the relatively small power generated by a microphone into which a man is speaking. The a.c. voltage generated by the microphone is applied to the grid of the triode, and a loud speaker is placed in the plate circuit instead of the resistance R of Fig. 414-2. With this arrangement,a d.c. plate current will be flowing through the loud speaker all the time, but no sound will be produced except when the magnitude of the plate current is made to fluctuate by the alternating voltage applied to the grid. The energy that operates the loud speaker therefore comes from the relatively powerful d.c. battery, but it is controlled by the grid. Thus a louder sound may be produced which has the same wave form and frequency as the original sound that entered the microphone.

A number of tubes connected together in sequence between a microphone and a loud speaker can be used to give an almost unlimited amount of amplification. In this way, it is possible to amplify the voice of a single individual so that it can literally be heard around the world, as is done in radio broadcasts.

416. The Use of Triodes as Oscillators. A triode may be used as an oscillator to generate alternating current, using the direct current battery in the plate circuit as the source of power. Thermionic tubes are widely used in this way, particularly to generate alternating currents of high frequency. The possibility of using a triode to generate alternating currents arises directly from its ability to act as a power amplifier, and a triode oscillator is nothing but a self-excited power amplifier. To illustrate this, let us consider again the arrangement discussed in the preceding section for amplifying alternating power from a microphone. The amount of power required to operate the microphone is very small compared to the output of the loud speaker. By diverting a small part of the output power back to the microphone, the loudspeaker can operate its own microphone without using up much of its output. Thus it is possible to have a self-sustained output of sound from the loud speaker. In other words, the triode with its accessories acts as an oscillator to produce sound vibrations.

Electric oscillations may be generated in a similar way without having a loud speaker to convert them into sound waves. The small fraction of the a.c. output voltage which is needed for self-excitation can be transmitted back to the controlling grid by electrical connections instead of using sound waves and a microphone. One arrangement which will generate electric oscillations in this way is shown in Fig. 416. This circuit differs from the circuit of Fig. 414-2 in that an inductive coil L_p takes the place of the load resistance R. Also a resonant circuit with a capacitance C and an inductance L is inserted in the line that leads to the control grid. Electric oscillations may occur in this resonant circuit, and once such an oscillation gets started, it will cause the grid to vary in potential as the charges surge back and forth. This variation in the potential of the grid will produce corresponding fluctuations in the plate current of the tube. A relatively large amount of power is available in this fluctuating plate current from the plate battery, and some of it can be fed back to maintain the oscillations in the resonant circuit attached to the grid. This can be done by placing the inductances L_p and L so that fluctuating lines of magnetic flux from the plate current in L_p will pass through L and induce an alternating emf in L.

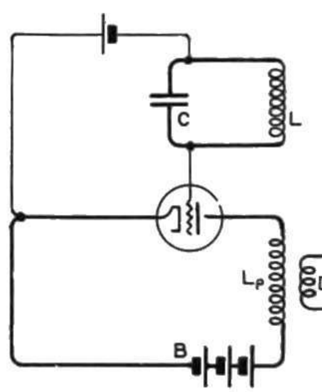


Fig. 416

With the arrangement shown in Fig. 416, the frequency of the oscillations generated will be the natural frequency of the resonant circuit which is used. Thus the desired frequency can be

obtained by a proper choice of the inductance L and the capacitance C. The initial electrical impulse required to start the oscillations in this circuit is always present. The thermal motion of electrons in the conductors, or any small electrical disturbance in the atmosphere may give such an impulse, and a circuit as shown in Fig. 416 will start to oscillate as soon as the connections are all made.

417. Multielectrode Tubes. Tubes with more than three electrodes are now most commonly used, but the essential function of such a tube serving as an amplifier is the same as that of a triode. For example, a pentode is a tube having a cathode and a plate with three grids between. To use such a pentode as an amplifier, two of the grids are connected to fixed potentials to serve secondary purposes. The first grid is used as a control grid with the cathode and plate so that these three electrodes function like a triode. In general, a pentode functioning as an amplifier has much larger values of μ and r_p than a corresponding triode. (See Sects. 422 and 423).

The purpose of one of the two extra grids in a pentode is to serve as a screen to prevent any undesired transfer of energy from the output back to the input. It will be recalled that such a transfer of energy may make a tube start to oscillate by itself, thereby interfering with its function as an amplifier. The small but definite capacitance between the plate and grid of a triode inside the tube constitutes an a.c. path through which energy may return from the plate circuit to the grid, and a screen grid inserted between the two can prevent this.

Tubes with more than five electrodes are generally composite tubes consisting of two or more simpler combinations in the same envelope. Thus a pentode and a diode may be inclosed in a single tube giving a total of seven electrodes.

418. Gas Filled Thermionic Tubes. Thermionic diodes and triodes are sometimes filled with gas or vapor, so that both thermionic electrons and gaseous ions contribute to the conduction. In general, a gas-filled tube will carry a larger current with a smaller potential drop than a highly evacuated tube of comparable size, but the current is subject to less control. For example, the grid of a gas filled triode can control the time at which a conducting discharge will start in the tube, but once the discharge starts the grid has no further control and the conductor cannot be stopped without reducing the plate voltage.

Chapter 42

THERMIONIC TUBES (continued)

420. Characteristic Curves of a Triode. The electric characteristics of a given triode may be described by a set of curves which show the current through the tube for any given combination of plate and grid voltages. Such a set of curves is shown in Fig. 420 for a common type of commercial triode. The plate voltage v_p and the grid voltage v_g are both measured relative to the cathode as indicated in Fig. 414-2. Note that any one curve of Fig. 420 is a graph of the plate current plotted against the difference in potential v_p between the plate and cathode for a given fixed value of the grid potential. Note that any one of these curves resembles the single curve which applies for a diode as was shown in Fig. 412 of this chapter.

421. Operating Voltages of a Triode. Before any voltage is applied to a triode to be amplified, it is necessary that the triode be in what may be called a suitable operating condition. This operating condition is established by applying certain fixed voltages to the grid and plate. These voltages serve as zero values to which other variable voltages may be added for amplification. The initial fixed values of voltage and the corresponding values of the current are commonly called quiescent values.

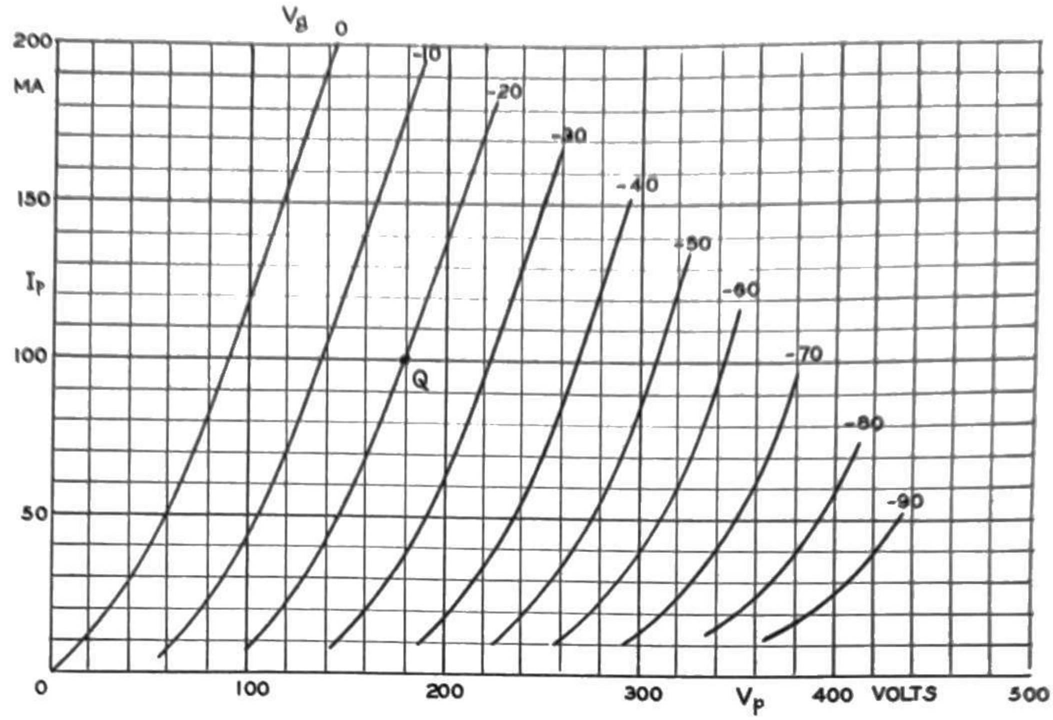


Fig. 420

A given operating condition may be represented by a single point on the diagram of characteristic curves. For example, the point Q on Fig. 420 represents a quiescent condition in which the grid voltage is -20 and the plate voltage is 180. This grid voltage is indicated by the marking at the top end of the sloping curve that passes through Q. The plate voltage is indicated by the scale at the bottom of the vertical line on which Q lies. The corresponding value of the plate current is seen to be about 100 milliamperes as indicated by the vertical scale at the left.

With a given starting point such as Q, the change in plate current that will be produced by a given change in plate voltage can be found from Fig. 420. For example, if v_p is increased from 180 volts to 200 volts while the grid voltage is kept constant at -20 volts, the current will increase from about 100 milliamperes to about 138 milliamperes. This new value of the current is found by moving along the sloping curve for which v_g has a constant value up to a point which lies on this curve and also on the ordinate for which v_p = 200 volts.

If we start again from the same starting point Q, the change in plate current produced by a given change in grid voltage can also be found from Fig. 420. Thus if v_g is increased from -20 volts to -10 volts, the current will change from about 100 milliamperes to about 180 milliamperes. This new value of the current is found by moving up along the ordinate for which v_p has a constant value of 180 volts to a point which lies on this ordinate and also upon the sloping curve for which v_g = -10 volts.

422. Plate Resistance of a Triode. As explained above, the characteristics of a given tube can always be described graphically by a set of curves like those given in Fig. 420. It is also possible to specify the characteristics of a tube by giving two numerical values which can be determined from the characteristic curves. One of these quantities is plate resistance r_p which will now be defined. The other is the amplification factor μ, which will be defined in the next section.

The plate resistance of a triode is defined by the equation

$$r_p = \frac{\Delta v_p}{\Delta i_p} \qquad (422\text{-}1)$$

where Δv_p is the increase in the plate voltage above the quiescent value and Δi_p is the corresponding increase of the plate current with a constant grid voltage. The resistance r_p is thus defined like the resistance of any path except that the potential drop and current are measured relative to the operating point as a zero.

The value of r_p may be determined from the curves of Fig. 420 since the reciprocal of r_p is equal to the slope of a curve of i_p vs. v_p. This follows from Eq. (422-1) which may be written in the form

$$\frac{1}{r_p} = \frac{\Delta i_p}{\Delta v_p} \tag{422-2}$$

Using the point Q as the operating point, we have already seen that i_p changes from 100 to 138 milliamperes when v_p changes 20 volts from 180 to 200 with v_g constant. Hence for this operating point,

$$\frac{1}{r_p} = \frac{\Delta i_p}{\Delta v_p} = \frac{38 \times 10^{-3} \text{ amp.}}{20 \text{ volts}} = .0019 \text{ amp. per volt} \tag{422-3}$$

Thus, $r_p = 526$ ohms (422-4)

423. Amplification Factor of a Triode. The current through a triode may be increased either by changing the grid voltage or by changing the plate voltage. The amplification factor μ is a measure of the relative effectiveness of these two changes as defined by the equation

$$\mu = \frac{\dfrac{\Delta i_p}{\Delta v_g} \text{ with } v_p \text{ constant}}{\dfrac{\Delta i_p}{\Delta v_p} \text{ with } v_g \text{ constant}} \tag{423-1}$$

The numerator of the above expression is the change in current per unit change in grid voltage alone, and the denominator is the change in current per unit change in plate voltage alone.

The value of μ may be found from a diagram of characteristic curves. For example, let us determine the value of μ at the operating point Q for the curves shown in Fig. 420. For that point, we have already noted that i_p increases from 100 to 180 milliamperes when v_g is increased ten volts from -20 to -10 volts with v_p constant at 180 volts. Thus

$$\frac{\Delta i_p}{\Delta v_g} = \frac{80 \times 10^{-3} \text{ amp}}{10 \text{ volts}} = .008 \text{ amp per volt.} \tag{423-2}$$

For the same operating point Q, we have from Eq. (422-3) in the preceding section that

$$\Delta i_p/\Delta v_g = .0019 \text{ amp per volt.} \tag{423-3}$$

Hence
$$\mu = \frac{.008 \text{ amp per volt}}{.0019 \text{ amp per volt}} = 4.2 \tag{423-4}$$

Since $\Delta i_p/\Delta v_g$ depends on the slope of the characteristic curves, and $\Delta I_p/\Delta v_g$ depends on how closely the curves are spaced, it follows that the values of r_p and μ will be the same for all locations of Q within an area where the characteristic curves are evenly spaced straight lines. Also the values of r_p and μ will be independent of the magnitude of Δv_p and Δv_g within this region.

424. General Equation for a Triode. The total change in current which results when the grid and plate voltage both change at the same time can be expressed in terms of r_p and μ. According to the definition of μ, a change in grid voltage Δv_g is μ times as effective as a change in

plate voltage Δv_p. Hence for simultaneous values of Δv_g and Δv_p, we may expect the effect on the current to be the same as would be produced by a larger $\Delta v_p'$ acting alone, provided

$$\Delta v_p' = \mu \Delta v_g + \Delta v_p. \tag{424-1}$$

If $\Delta v_p'$ did act alone, Eq. (422-1) above requires that the change in current Δi_p would be given by

$$\Delta i_p = \frac{\Delta v_p'}{r_p}. \tag{424-2}$$

Substitution from Eq. (424-1) in Eq. (424-2) then gives

$$\Delta i_p = \frac{\mu \Delta v_g + \Delta v_p}{r_p} \tag{424-3}$$

for the general case where Δv_p and Δv_g occur simultaneously.

425. Voltage Amplification by a Triode. When a triode is used as an amplifier, the voltage to be amplified is applied to the grid as an increment Δv_g superimposed upon the quiescent grid voltage v_g. For example Δv_g might be one of the peaks of alternating voltage generated by a microphone in the grid circuit. If there is to be any amplification, this input voltage change must produce a larger output voltage change Δv_R across a loud speaker or other resistance R connected in the plate circuit as shown in Fig. 414-2. The resultant voltage amplification is defined as the ratio of the output voltage change to the input voltage change.

The amount of amplification which will be produced by a given triode can be computed in terms of the resistance R of the load and the constants μ and r_p of the triode. An input voltage change Δv_g will result in a change of plate current ΔI_p through the load R. Hence the output voltage will change by an amount $R\Delta I_p$. This increase in the potential drop must be accompanied by a decrease in the potential difference v_p across the triode because the load R and the triode are connected in series with the constant emf of the plate battery. This decrease in plate voltage across the triode will tend to counteract the effect of the increased grid voltage, so that the resultant change in plate current will be less than what would be expected due to the change in grid voltage alone. To compute the actual amplification we may apply Eq. (424-3), with Δv_p being a decrease in voltage equal to the increase $R\Delta I_p$ which appears across the load resistance. Thus we may write

$$\Delta I_p = \frac{(\mu \Delta v_g) + (- R\Delta I_p)}{r_p} \tag{425-1}$$

Since the over-all amplification A is the ratio of the voltage increment across the load (R I_p) to the input voltage Δv_g, we may write

$$A = \frac{R\Delta I_p}{\Delta v_g} \tag{425-2}$$

Substitution from Eq. (425-1) then gives

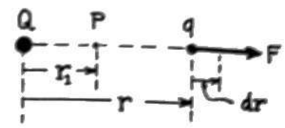
$$A = \mu \frac{R}{(R + r_p)} \tag{425-3}$$

The value of $R/(R + r_p)$ will always be less than unity, and so voltage amplification A will always be less than the amplification factor μ of the tube.

Appendix A

DERIVATION OF $V = K_0 Q/r$ BY CALCULUS

To get the potential due to Q at a point P a distance r_1 from Q, it is necessary to compute the work that can be done by the repulsive force F on a movable charge q as q moves from P to an infinite distance. Since F decreases as r increases, it will be desirable to split the path up into small steps each of size dr. The work dW done in any one step will then be $F\, dr \cos\theta$, where θ is the angle between F and dr. The total work W will then be the sum of these terms, and it may be expressed by the integral



$$W = \int_{r_1}^{r_2} F\, dr \cos\theta \qquad (1)$$

In moving away from Q by any path, we may choose our steps so that θ is either $0°$ or $90°$. The steps with $\theta = 90°$ will give zero contribution and hence may be disregarded. For the steps with $\theta = 0°$, $\cos\theta = 1$. Hence, we may write

$$W = \int_{r_1}^{r_2} F\, dr \qquad (2)$$

Since $F = K_0 Qq/r^2$ where Q and q are constants, equation (2) may be written

$$W = K_0 Qq \int_{r_1}^{r_2} \frac{dr}{r^2} \qquad (3)$$

Integrating

$$W = -K_0 Qq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]. \qquad (4)$$

If r_2 is infinity, $1/r_2 = 0$ and

$$W = K_0 \frac{Qq}{r_1} \qquad (5)$$

The potential V at P is the work that can be done per unit charge. Hence, using $V = W/q$ we obtain

$$V = K_0 \frac{Q}{r_1} \qquad (6)$$

APPENDIX B

As far as magnetic effects are concerned, a charge q moving with a velocity v is equivalent to a current I flowing in a wire of length L provided $qv = IL$. This equivalence may be easily shown by choosing I and L so that the total amount of moving charge in the length of wire L is equal to q. Consider that this charge is distributed along the wire and moving with a velocity v like a line of cars moving in a one-way street. The time t required for all the charges in a line of length L to pass a given point will be L/v. The current I past that point will be q/t. Combining these two expressions gives

$$IL = qv \qquad (1)$$

Appendix C

DERIVATION OF EQUATION FOR THE FIELD OF A STRAIGHT WIRE

The field at a point P a distance R from the straight piece of wire carrying a current can be found by applying Ampere's law as follows. That part of the field due to the segment dL as indicated in Fig. 1 will be

$$dB = b\frac{I\,dL\,\sin\theta}{r^2} \tag{1}$$

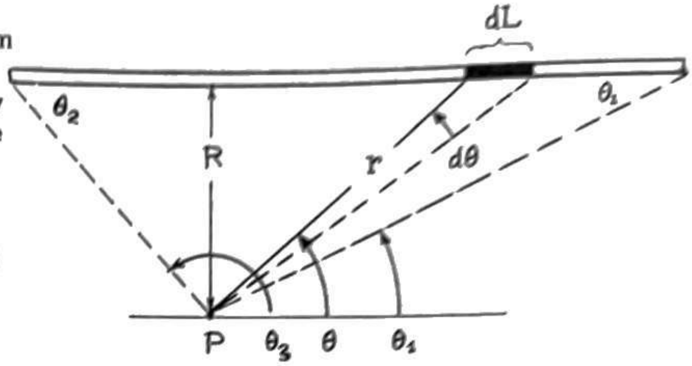


Fig. 1

This expression may be conveniently integrated with θ as the variable. In order to do this, the differential dL and the variable r must both be expressed in terms of θ and constants. We can write $r\,d\theta/dL = \sin\theta$ so that

$$dL = \frac{r\,d\theta}{\sin\theta} \tag{2}$$

Also we may write $R/r = \sin\theta$, so that

$$r = \frac{R}{\sin\theta} \tag{3}$$

Substituting from Eqs. (3) and (2) in Eq. 1, to eliminate r and dL we obtain

$$dB = b\frac{I}{R}\sin\theta\,d\theta. \tag{4}$$

Integration from $\theta = \theta_1$ to $\theta = \theta_3$ then gives

$$-B = b\frac{I}{R}\left[\cos\theta\right]_{\theta_1}^{\theta_3} \tag{5}$$

or

$$B = b\frac{I}{R}(\cos\theta_1 - \cos\theta_3) \tag{6}$$

The angle $\theta_2 = 180 - \theta_3$, so that

$$B = b\frac{I}{R}(\cos\theta_1 + \cos\theta_2) \tag{7}$$

Appendix D

FORCE BETWEEN TWO MAGNETIC POLES

To prove that two concentrated magnetic poles p_1 and p_2 a distance r apart will exert a force F on each other according to the equation

$$F = \frac{1}{4\pi\mu}\frac{p_1 p_2}{r^2} \tag{1}$$

let us temporarily remove p_2 from the point P and replace it with a short segment of wire carrying a current as shown in Fig. 1. If dL is the length of the wire, the field H_1 acting on p_1 due to the current I will be given by

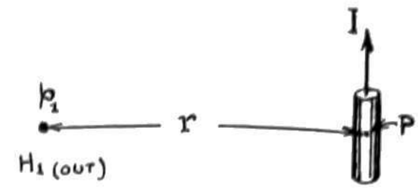


Fig. 1

$$H_1 = \frac{1}{4\pi}\,\frac{dL\,I}{r^2} \tag{2}$$

This force will be directed out from the plane of the drawing. Hence the force F acting on p_1 will be given by

$$F = \frac{p_1}{4\pi}\,\frac{dL\,I}{r^2} \tag{3}$$

Now according to the universal law of reaction, there must be an equal and opposite force acting on the segment of wire down into the drawing. From this we know that the segment of wire must be in a flux density B directed from p_1 to dL such that

$$B = \frac{F}{I\,dL} \tag{4}$$

This flux density B at P must be due to p_1 and hence the corresponding value of H at P due to p_1 will be B/μ. Substitution from Eq. (4) then gives

$$H = \frac{F}{\mu I\,dL} \tag{5}$$

If the wire is removed and p_2 is replaced at P, the force F on p_2 will be equal to

$$F = p_2 H \tag{6}$$

Combining Eqs. (3), (5), and (6), then gives

$$F = \frac{p_1 p_2}{4\pi\mu r^2} \tag{7}$$

PROBLEMS

GROUP 1

10. Two small bodies charged in the same way so that their charges are equal repel each other with a force of one newton when they are two meters apart. What is the charge on each body expressed in microcoulombs?

Solution: Use the equation, $1 \text{ newt} = (9 \times 10^9) \ \frac{\text{newt}}{\text{coul}^2/\text{m}^2} \ \frac{q^2}{4\text{m}^2}$

$$q^2 = \frac{4}{9 \times 10^9} \text{ coul} = 4.44 \times 10^{-10} \text{ coul}$$

$$q = 2.1 \times 10^{-5} \text{ coul} = 21 \text{ microcoulombs}$$

11. Two small bodies, charged in the same way so that their charges are equal, repel each other with a force of .0081 newt when .2 m apart. What is the charge on either one?

12. Find the force which a charge of 800 microcoulombs will exert on another charge of 30 microcoulombs 6 meters away.

13. If two bodies having a charge of 1 coulomb each were 10 meters apart, what would be the repulsive force between them? Express in newtons, in grams, and in tons. (1 newt = .000112 ton)

14. A charge B of $+5\mu$ coul is 8 m east of a charge A of $+2\mu$ coul, and a third charge of -3μ coul is placed half way between the two. What force acts on A?

15. A piece of copper wire 1 mm in diameter and 1 cm long contains approximately 6.6×10^{20} atoms. (a) How much charge would have to be removed from the wire to remove one electron from each atom? (b) If these electrons as a group could be moved to a point one tenth of a mile (161 meters) from the piece of copper wire, leaving the copper with an equal positive charge, what would be the force between the two in tons. (1 newt = .000112 tons)

GROUP 2

20. The force on a charge of 80 microcoulombs at a certain point in space is 0.4 newtons east. What is the field strength?

21. The force on a charge of -60 microcoulombs is 0.12 newtons north. What is the magnitude and direction of the field strength?

22. A charged body weighing 2 grams can be supported by an upward field of 5000 newtons per coulomb. What is the charge on the body?

23. Show that $\mathcal{E} = K_0Q/r^2$ follows from Coulomb's law and from the definition of $\mathcal{E}$ as F/q.

24. Explain how the relationship between $\mathcal{E}$ and q in $\mathcal{E} = F/q$ differs from the relationship between $\mathcal{E}$ and Q in $\mathcal{E} = K_0Q/r^2$.

25. A fixed charge of +40 microcoulombs is 10 m east of a fixed charge of -20 microcoulombs. (a) Find the field strength at a point 5 m from each charge, expressed in newt/coul. (b) Find the field at a point 10 m west of the smaller charge.

26. A charge B of +200 microcoulombs is 10 m east of a charge A of -200 microcoulombs, and a third charge C of 50 microcoulombs is 10 m from each of the others and directly north of a point halfway between A and B.
(a) Find the total field strength at C due to A and B in newt/coul.
(b) Find the total force on C due to A and B.

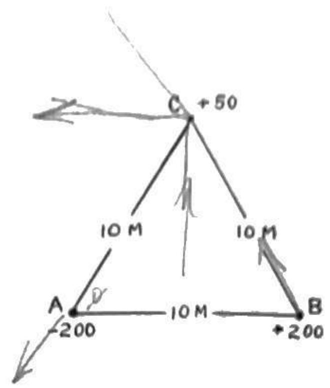


Fig. 26

27. (a) If Q is a fixed concentrated charge of 2×10^{-9} coulomb, what will be the field due to Q at a point 1 meter away, and how many lines of force would be drawn through a sphere 1 m in radius (with its center at Q) in order to represent the field in newtons per coulomb at all points on the spherical surface?
(b) Repeat for a point 3 meters away and for a sphere having a radius of 3 meters, and compare the answers with those of part (a).

GROUP 3

30. What work must be done to carry a positive charge of 10 coulombs from a place where the potential is 80 volts to a place where it is 150 volts?

31. It takes 400 joules of mechanical work to carry +5 coulombs of charge from a point A to a point B. What is the difference in potential between A and B, and which point is at the higher potential?

31x. A potential difference of 100 volts exists between A and B, and it takes 600 joules of mechanical work to carry a positive charge q from B to A. (a) What is the magnitude of q? (b) If one coulomb of charge has 800 joules of electrostatic potential energy at A, how much such energy will a coulomb have at B?

2, 32. (a) A small ball rolls on a level insulating track without friction from a point A to a point B where A is maintained at a potential 50,000 volts higher than B. If the ball has a charge of .5 microcoulomb, how much kinetic energy will it acquire?
(b) What will be the average force in grams if A and B are 2 meters apart?
(c) What is the average field strength along the track expressed in volts/meter and in newtons/coulomb?

33. (a) How much work will be required to move 50 coulombs a distance of 30 meters along a path where the field is 400 volt/meter in a direction opposed to the motion?
(b) What is the average force on the 50 coulombs, expressed in newtons?

GROUP 4

40. Compute the potential due to a charge of 1 coulomb at a point 3000 meters away, and show by a substitution of units how the answer may be obtained in volts.

41. Find the potential at a point 2 meters away from a charge of 4 microcoulombs.

42. Two charges of +150 microcoulombs each are 10 meters apart.
(a) What is the potential at a point P half way between the two charges?
(b) What is the field strength at P?

42x. A charge of +150 microcoulombs is 10 meters from an equal negative charge.
(a) What is the potential at a point P half way between the two charges?
(b) What is the field strength at P?

43. Two positive charges Q_1 and Q_2 of 5 microcoulombs each are 6 meters apart as shown in Fig. 43.
(a) What is the potential at each point A, B, and C due to both charges Q_1 and Q_2?
(b) What is the difference in potential between A and B, and in what direction would the average force act on a movable positive charge along a path between A and B?

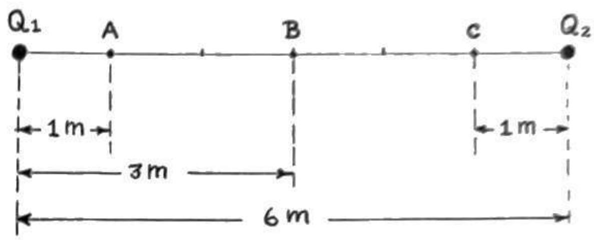


Fig. 43

43x. Solve the preceding problem taking Q_1 to be -5 microcoulombs and Q_2 to be +5 microcoulombs.

44. A charge of 24 microcoulombs is at A and one of 20 microcoulombs is at B as shown in Fig. 44. What is the potential at C in volts?

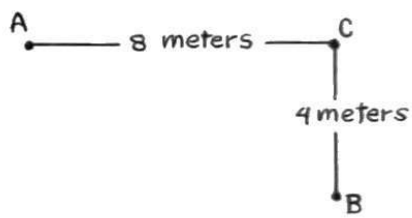


Fig. 44

45. A point is at a negative potential due to a fixed charge Q nearby. In what direction would a movable negative charge tend to move relative to Q?

GROUP 5

50. (a) Explain why there can be no field inside the material of a conducting body when the charges come to rest.
(b) Explain why all parts of a conductor must be at the same potential if no electric field can exist between any two points in the material.

51. (a) Is it possible for a body to be at a potential other than zero if the total charge on the body is zero? Explain.
(b) Is it possible for a body to have a charge and still be at zero potential? Explain with an example.

52. An electroscope is charged positively. As a large negative charge is brought up from a great distance, the leaf first collapses and then deflects increasingly as the negative charge is brought closer. Explain what has happened in terms of the equation $V_1 = aQ_1 + bQ_2$.

53. In Prob. 52, how would the value of b be changed if the charge Q_2 were brought closer to Q_1?

54. A belt type electrostatic generator transports .0002 coulombs of charge from the low potential terminal to the high potential terminal each second. (a) If the difference in potential is 2,000,000 volts, what power in watts is required to run the belt as far as electrostatic forces are concerned? (b) If the belt runs 4 meters per second, what electrostatic force acts on the side of the belt which carries the charge? Express in newtons and pounds.

55. A metal ball M as shown in Fig. 55 has been charged with a positive charge Q_1, giving it a potential of 1000 volts when it is not close to any other bodies. A hollow conducting shell S is charged with a positive charge Q_2 to a potential of 100,000 volts.
(a) If M is inserted in the opering of S without touching S, will the potential of M be (A) 1000 volts, (B) more than 1000 but less than 100,000 volts, (C) 100,000 volts, (D) more than 100,000 volts. Give reason for your choice.
(b) If there is any increase in the energy of the charge on M, explain where it came from.
(c) If M is touched to the inside of S, and then removed far from S, how would the potentials of M and S compare with their respective original values? Explain.

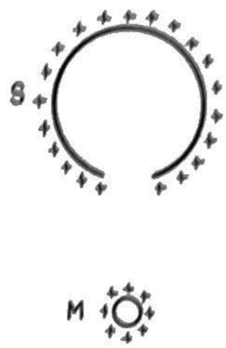


Fig. 55

56. If the potential of S in the above problem is increased by repeatedly charging M to a potential of 1000 volts and touching it to the inside of S, which of the following factors might theoretically limit the potential that could be reached on S: (A) Amount of force available to move M, (B) leakage of charge from S through the air, (C) limitation of starting potential of M to 1000 volts.

57. (a) What would be the kinetic energy of an electron shot from a gun by moving through a potential difference of 5000 volts? Express in joules.
(b) Compute the velocity of the electron in part (a), assuming that the kinetic energy is given by $mv^2/2$.

58. A gun in a television tube consists of a filament F and two parallel plates A and B with holes as shown in cross-section. Electrons emitted from F are accelerated towards A and some of them will pass on through the holes in A and B, emerging with a velocity which will depend upon the potentials V_0 and V_1. It is to be assumed that an electron passing through a hole in a plate is so near to the plate that it must have reached the same potential that it would have reached it is actually hit the plate.
(a) If A is 8000 volts higher in potential than F, and B is 2000 volts higher than A,

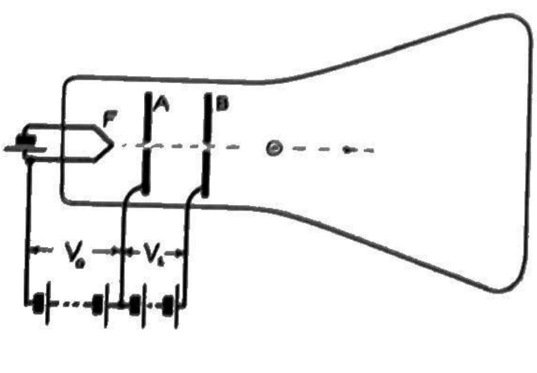


Fig. 58

how many joules of kinetic energy will an electron have when it reaches A?
(b) How many joules of kinetic energy will a electron have after it gets past B?
(c) Give the direction of the force on the electron between F and A, and between A and B.

59. Repeat the preceding problem for the case where B is 2000 volts lower in potential than A.

GROUP 6

60. A body having a mass, m, of 0.1 gm is to be supported at rest in air between two horizontal plates by a uniform electric field, $\mathcal{E}$. The body has a charge, Q, of .004 microcoulombs, and the distance between the plates is 0.5 cm. What difference in potential will be required?

60x. Find the mass of an oil drop floating between two horizontal metal plates 0.8 cm apart if the top plate is 2000 volts higher in potential than the lower plate, and if a charge of 1 electron is just enough to support it.

61. Two parallel plates each having an area of 2 square meters are .005 meters apart. If a charge of .001 μ-coul is removed from one plate and placed on the other, what will be the field between the plates and what will be the difference in potential between the two plates?

62. If the two plates of the preceding problem are moved apart so that the distance between them is doubled, what effect will that have on the field and on the potential difference?

63. Explain the difference between the situation for which $\mathcal{E} = K_0Q/r^2$ holds and that for which $\mathcal{E} = K_0 4\pi Q/A$ holds.

GROUP 7

70. A charge of .002 coulomb transferred from one plate of a condenser to the other produces a difference in potential of 500 volts. Compute the capacitance in mfd.

71. A condenser is marked 4 mfd, 400 volts.
(a) Can the condenser be used satisfactorily with a difference in potential of 200 volts, and if so, what will be its capacitance when charged to 200 volts?
(b) What charge must be transferred to charge this condenser to 200 volts?

72. (a) Compute the capacitance of a condenser made of two parallel plates .3 cm apart in air where each plate has an area of 2 square meters.
(b) What would be the capacitance of these two plates if they were immersed in oil so that the space between was filled with oil having a dielectric constant of 2?

72x. Compute the capacitance of a condenser made of two parallel plates 20 cm x 20 cm square separated by a sheet of mica .2 mm thick. (For mica, k = 6.0)

73. The capacitance of two metal plates 2 mm apart in air is 500 mmfd. What will be the capacitance if they are separated to a distance of 4 mm and a glass plate of that thickness is inserted between the plates? The dielectric constant k of glass is 5.

74. (a) A parallel plate condenser has a capacitance of 500 mmfd when the plates are 3 mm apart. If it is charged to a potential difference of 800 volts, what will be the charge on the condenser?

(b) If the charged plates are moved until they are 12 mm apart, without adding or removing any charge on either plate, what will be the new potential difference and the new capacitance?

75. One plate of a parallel plate condenser is permanently connected to the central terminal of an electroscope, and the other is permanently connected to the case. Flexible leads are used so that the plates can be moved without disturbing the connections. The electroscope has a fixed capacitance of 20 mmf. When the condenser plates are close together so that the condenser alone has a capacitance of 10,000 mmf the condenser and the attached electroscope are charged to a potential difference of 1.5 volts by a temporary connection to a dry cell. If the condenser plates are then moved apart until the capacitance of the condenser is reduced to 30 mmf, what will be the potential difference indicated by the electroscope?

76. A condenser has a charge of 1000 microcoul. When charged to a potential difference of 100 volts, what is the energy of the charged condenser?

77. A condenser with a capacitance of 2 microfarads is charged to a potential difference of 6000 volts. What energy does it have?

78. (a) A variable radio condenser having a capacitance of 500 mmfd is charged to a potential of 100 volts. What is the energy of the charged condenser?

(b) If the plates are separated to reduce to capacitance to 100 mmfd while they are insulated so that there can be no change in the charge, what will be the energy of the condenser?

(c) How can this change of energy be accounted for?

GROUP 8

80. If the current passing a point is .03 milliamperes,
(a) Express its value in amperes and in microamperes,
(b) Compute the number of electrons passing the point in one second, and
(c) Compute the time in minutes required for one coulomb of charge to pass.

GROUP 10

100. A current of 10 amperes passing through an electric heater for 1000 seconds gave out 50,000 joules of heat.
(a) Assuming none of the electrical energy was expended except to produce heat, what was the difference in potential V between the terminals of the heating element?
(b) How much power was expended?

100x. (a) A current of 50 amperes passing through a conductor for ten seconds gave out 100 joules of heat. What was the potential difference?
(b) What power was expended in the conductor?

101. A charge of 120 coulombs flows through a wire from A to B in 2 minutes when A is maintained 3 volts higher in potential than B by a chemical cell. How much heat in calories will be produced between A and B in that time?

102. A straight wire lies between points A and B 50 cm apart. A current of 2 amp flows from A to B, and the potential difference between the points is 10 volts. Find the average force on a coulomb of charge between A and B, and find the average electric field strength between A and B. State direction.

GROUP 11

111. For how many seconds must a current of 10 amperes flow to deposit 500 gm (about 1 lb) of aluminum?

112. If W lbs of copper are deposited by a current of 4 amperes in 500 seconds, how much copper (expressed in terms of W) will be deposited by a charge of 500 coulombs passing through the cell?

113. A current I flows for a time t and deposits 8 grams of copper. How much silver would be deposited by the same current in the same time?

114. If a current of 2 amperes flowing for 5 minutes deposits .408 gm of a metal, what is its electrochemical equivalent?

115. Compute the electrochemical equivalent of zinc using the values given for its atomic weight, its valence, Avogadro's Number N, and the electronic charge e.

116. (a) If 2 grams of hydrogen occupies 22.4 liters under standard conditions of temperature and pressure, what volume of hydrogen will be liberated by a current of 1 ampere flowing for 1 hour?
(b) What volume of oxygen will be liberated at the same time at the other electrode?

117. A current of 5 amperes flows through acidulated water until 16 gm of oxygen are liberated at the anode.
(a) How many grams of hydrogen are liberated at the cathode?
(b) Under the same conditions of pressure and temperature, how does the volume of hydrogen compare with the volume of oxygen? Explain.

118. A current of 3 amperes is passed for 20 minutes through an electrolytic cell with nickel ions being deposited on the negative electrode.
(a) How many ions are deposited?
(b) What is the mass of an ion of nickel?
(c) What is the total mass of nickel deposited in the 20 minutes?
(d) What is the electrochemical equivalent of nickel?

119. If a current I flowing for a time t transports 96,500 coulombs through an electrolytic cell, what mass of chromium would be deposited?

GROUP 12

120. A dry cell has an emf of 1.5 volts.
(a) How much work is done by the cell when a current of 0.5 ampere is furnished for two hours? Express in joules and in calories.
(b) What transformation of energy is taking place in the cell?

120x. A generator having an emf of 110 volts furnishes 60 amperes of current for fifteen minutes.
(a) How much electrical energy will be produced in the generator during the 15 minutes?
(b) Where does this energy come from?
(c) What electrical power is generated?

121. Find the number of joules in a kilowatt-hour.

122. A generator which generates an emf of 120 volts is furnishing a current of 50 amperes. What electrical power will it deliver? Express in kilowatts and in h.p.

123. A variable radio condenser is permanently connected to a battery so that the potential difference between the plates will be maintained constantly by the battery at 200 volts.
(a) When the condenser is set to have a capacitance of 100 mmfd, what energy will be stored in the condenser?
(b) If the capacitance is increased to 300 mmf, how much additional charge must the battery transfer to keep the potential constant?
(c) How much energy does the battery expend in furnishing this charge?
(d) How much energy will be stored in the condenser after the increase in capacitance, according to the equation $W = Q\,V/2$?
(e) Are the answers to (a), (c), and (d) consistent with the principle of conservation of energy?

GROUP 13

130. An electric iron is labeled 110 volt, 6 amp. What is the resistance of the heating element?

131. What is the drop in potential across a resistance of 200 ohms if .3 amp is flowing through it?

15

GROUP 14

140. A current of 10 amperes was drawn from a 110 volt line to operate a stove for 1 hour.
(a) How many coulombs of charge passed through the stove?
(b) How much energy was used? Express in joules, in calories, in kilowatt-hours.
(c) How much power was used?
(d) What is the resistance of the stove?

141. (a) Find the resistance of the heating element of a 110 volt, 600 watt waffle iron and the resistance of a 110 volt, 50 watt lamp.
(b) What is the current in each?

141x. Find the resistance and current for a 100-watt, 110-volt lamp bulb at the operating temperature.

142. How much current will a 150-watt electric soldering iron take to heat it properly if it is designed for use on a 32-volt country lighting system?

143. A 60-watt, 120-volt soldering iron is connected to a 6-volt battery. At what rate will heat be produced? (Express in watts)

144. A current of 10 amperes is flowing through a long wire having a resistance of .5 ohm. (a) What is the difference in potential between the two ends of the wire? (b) Will the amount of energy lost in heat by each coulomb passing through the wire depend on the number of coulombs passing through the wire per second? Explain.

145. A 100-watt lamp was operated for 10 hours.
(a) How much power did it require?
(b) How much did it cost to operate it if the electricity cost $.05 per kilowatt-hour?

145x. Compute the cost of lighting a home for 3 hours if four 25-watt lamps, five 60-watt lamps, and two 100-watt lamps are used, where there is a difference in potential of 120 volts across each bulb, and where electric power costs $.05 per kwh.

146. A current of 5 amperes flows through a resistance of 20 ohms for two minutes.
(a) How much power is dissipated? (b) How much heat is produced?

147. A lamp bulb designed for operation on 120 volts takes a current of .1 amp when 10 volts difference in potential is applied. Should the bulb be rated 144 watts, or more than 144 watts, or less than 144 watts? Give reason for your answer.

148. Compute the amount of power expended in an x-ray tube carrying a current of 5 milliamperes through a difference in potential of 100,000 volts.

149. (a) Prove that if a number of resistive heating devices are all designed to operate at the same voltage, the power consumed by any one will be inversely proportional to its resistance. (b) Prove that if the same current passes through several resistive heating devices in series, the power consumed by any one will be directly proportional to its resistance.

GROUP 15

150. (a) In Fig. 150, V_1 = 12 volts, and V = 14 volts when the current is .5 amp. Find the value of R_2.
(b) Express the ratio V_1/V_2 in terms of R_1 and R_2.

150x. In Fig. 150, V_1 = 1 volt, V_2 = 4 volts, R_2 = 20 ohms. Find R_1.

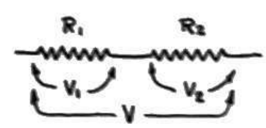


Fig. 150

151. An electric heater is connected in series with a silver plating cell so that the charge Q passing through can be determined by the mass of silver deposited. A potential difference V of 110 volts is applied to the combination and 120,000 joules of heat H_1 are given out in 10 minutes in the heater while 1200 coulombs of charge pass through.

(a) What is the difference in potential V_1 across the heater?
(b) How much energy is given out in the plating bath?
(c) Express the difference in potential V_2 across the plating bath as a function of Q, H_1, and V.

152. The difference in potential across the lamp in Fig. 152 is 6 volts. The total current I is 10 amperes. What is the value of the resistance R, if the current in the lamp is 4 amperes?

Fig. 152

153. In Fig. 153 the difference in potential V_1 is 28 volts. V_2 is 24 volts. Find the current in the 4 ohm resistance and in the resistance X.

154. An electric toaster designed to operate at 400 watts on a 32-volt line is to be operated at its rated power on a 110 volt line. (a) Explain how this could be done with the aid of one other resistor, and specify the resistance required. (b) What percent of the power taken from the 110 volt line would appear in the toaster?

Fig. 153

155. A 55-watt 110-volt soldering iron and a 220-watt 110-volt soldering iron are connected in series to a 220 volt line.
(a) Compute the current needed in each if it is to operate satisfactorily at its proper temperature.
(b) Could the two irons be satisfactorily operated on a 220-volt line in this way?
(c) How would the rate of heat production in the smaller 55-watt iron compare with the rate of heat production in the larger 220-watt iron?

156. (a) If the two soldering irons of problem 155 are connected in parallel to a 110-volt outlet, in which iron will more heat be generated?
(b) Will the operation of the irons be satisfactory?

GROUP 16

160. In Fig. 160, the difference in potential between A and B is 43 volts. Find the equivalent resistance of the network between A and B.

161. A total current of 3.6 amperes flows through a combination of three resistances connected in parallel. The three resistances have values of 1, 3, and 6 ohms, respectively.
(a) What is the current in each resistance?
(b) What is the potential difference between the terminals of the combination?
(c) What is the resistance of the combination?

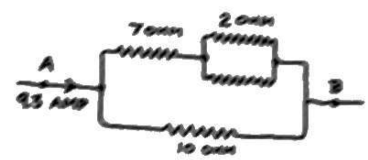


Fig. 160

161x. Three resistors of 2, 5, and 10 ohms, respectively, are connected in parallel and the current flowing through the combination is 16 amperes. Find the equivalent resistance and the current in each branch.

162. Find the equivalent resistance of the network shown in Fig. 162a if the external connections are made to bring the current in at A and take it away at C.

Solution: First reduce the network to the more simple network of Fig. 162b by replacing the parallel combination of 2 and 5 ohms with a single equivalent resistance of 10/7 ohms. The network can then be further simplified by noting on Fig. 162b that the 8 ohm resistance and the 10/7 ohm resistance form a series combination having an equivalent resistance 66/7 ohms as indicated in Fig. 162c. Finally note that the two resistances of Fig. 162c constitute a parallel combination with an equivalent resistance of 66/73 ohms, which is .905 ohms.

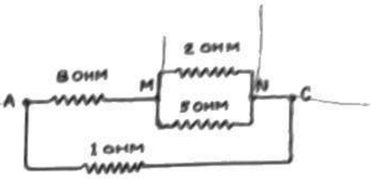


Fig. 162a

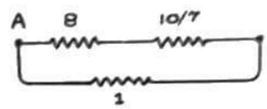


Fig. 162b

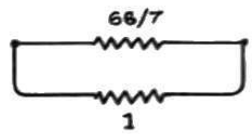


Fig. 162c

163. (a) Find the equivalent resistance of the network of the preceding problem if connections are made to M and N.
(b) If a difference in potential of 45 volts is applied between M and N by external connections not shown, how much current will flow through the 8 ohm resistance.

164. (a) Compute the equivalent resistance between A and B for the network shown in Fig. 164.
(b) Compute the equivalent resistance between M and N.

165. A 2 ohm, a 3 ohm, and a 6 ohm resistance are available for use, singly, two at a time, or three at a time, in any combination desired. Give the smallest and the largest resistance that could be obtained, stating what resistances would be used and showing how they would be connected.

166. (a) A current is flowing through a resistance R of 1 ohm between two points A and B. If a 1000 ohm resistance is added in parallel to R, what will be the per cent change in the resistance between A and B? Will the change be an increase or a decrease?
(b) Repeat (a) for a 1000 ohm resistance added to R in series between the points A and B.

167. Starting with the equation $1/R = 1/R_1 + 1/R_2 + \ldots\ldots\ldots$, show that if a number of resistances are connected in parallel, the combined resistance will be smaller than the smallest individual resistance.

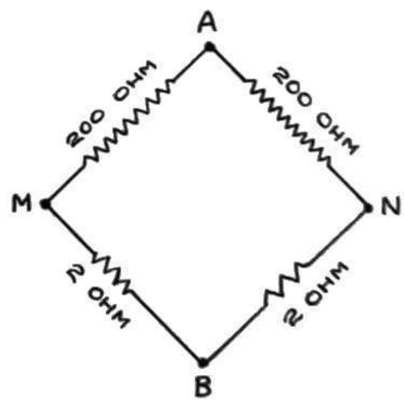


Fig. 164

GROUP 18

180. A galvanometer having a resistance of 200 ohms is deflected 1 division by a current of 10^{-6} amp. How much resistance should be used as a shunt to make it into an ammeter reading 2 amperes for each division?

180x. A galvanometer having a resistance of 12 ohms and a full scale deflection for 0.02 amp is to be made into an ammeter with a full scale deflection of 0.5 amp, using a single added resistance. Show on a diagram how this can be done and compute the exact value of the added resistance.

181. A galvanometer of resistance 90 ohms and sensitivity 10^{-2} amperes per division is made into an ammeter by a shunt of resistance 10 ohms.
(a) What is the resistance of the ammeter?
(b) What is the current through the ammeter when the deflection is 5 divisions?

182. A milliammeter having a sensitivity of 1 milliampere per scale division and an over-all resistance of 2.00 ohms contains an internal shunt having a resistance of 2.20 ohms. What must be the resistance of an external shunt so that the total current flowing through the combination will be 5 milliamperes per scale division on the milliammeter?

GROUP 19

190. A galvanometer having a resistance of 180 ohms is deflected 1 division by a current of 0.4×10^{-6} amp. What additional resistance is needed and how should it be connected to make a millivoltmeter reading 10^{-3} volts per division?

190x. A galvanometer has a coil resistance of 50 ohms, and a current of 0.001 amp causes it to deflect one scale division. Calculate the resistance required to change the galvanometer into a voltmeter reading 10 volts per scale division, and show on a figure how it should be connected.

191. A voltmeter is connected in series with a resistance R and a potential difference of 110 volts is applied to the extremes of the combination. The voltmeter has a resistance of 200 ohms and reads 5 volts when connected as stated. Compute the value of R.

192. (a) A voltmeter and an ammeter are connected to a lamp R as shown in Fig. 192, to determine the resistance of the lamp by the use of Ohm's law. The ammeter reads 0.1 amp and the voltmeter reads 20 volts. What is the resistance between the points A and B?
(b) If the voltmeter has a resistance of 400 ohms, what is the value of the resistance R?
(c) If the voltmeter had a resistance of 100,000 ohms instead of 400 ohms, what would it read when the ammeter read 0.1 amp?

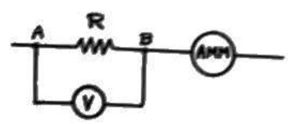


Fig. 192

193. A milliammeter having a resistance of 20 ohms reads 5 milliamperes at full scale deflection. How could this meter be used as a millivoltmeter having a full scale deflection of 100 millivolts?

194. A multiplier is used with a voltmeter having a full scale reading of V_0 so that a larger difference in potential V across the combination corresponds to a full scale reading on the voltmeter. If R_V is the resistance of the voltmeter and R_1 the resistance of the multiplier, express R_1 in terms of R_V, V_0, and V.

195. A certain microammeter has a full scale reading of .0005 amp and a resistance of 120 ohms. It is shunted by a resistance of 120 ohms and the combination is connected in series with a resistance of 20,000 ohms between two terminals A and B. What difference in potential between A and B will be indicated by a full scale reading of the microammeter?

GROUP 20

200. A current of 5 amp is flowing through a number of cells as shown in Fig. 200.
(a) If the cells are storage cells, state for each battery whether it is being charged or discharged.
(b) Find the difference in potential between A and B and state which point is higher.
(c) Find the difference in potential between B and C and state which is higher.

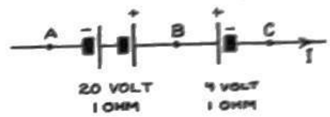


Fig. 200

200x. Repeat the parts of problem 200 for the cells shown in Fig. 200X. (The current is driven in this direction by some strong source of emf in the rest of the circuit which is not shown.)

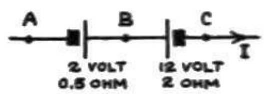


Fig. 200X

Solution: (a) The cell AB is being discharged, and the cell BC is being charged. In the first cell the chemical emf is in the direction of the current, and is therefore doing work. In the second cell, the charges move in at the high potential end, do work against the emf, and move out at the low potential end with less energy. The work done against the emf is stored in the cell as chemical energy.

(b) For a cell with current in the direction of the emf the gain in potential $V = E - IR_i$. That is, charges gain energy from the emf and lose some as heat. Hence

$$V = 2 \text{ volts} - (5 \text{ amp} \times .5 \text{ ohm}) = (2 - 2.5) \text{ volt} = -.5 \text{ volt}$$

Hence the charges gain less than they lose and B is <u>lower</u> in potential than A, or A is higher than B.

(c) For a cell being charged, the loss in potential $V = E + IR$. That is, the charges lose energy to the chemical emf and also as heat. Hence

$$V = 12 \text{ volts} + (5 \text{ amp} \times 2 \text{ ohms}) = (12 + 10) \text{ volts} = 22 \text{ volts}$$

Since this is a loss in potential, B is higher in potential than C.

201. What condition would be necessary in order for the positive terminal of a cell to be at a lower potential than the negative terminal if the cell has an emf of 1.5 volts and an internal resistance of .5 ohm?

GROUP 21

210x. A battery of storage cells furnishes an emf of 100 volts and has an internal resistance of 6 ohms. It is connected in a circuit with three external resistances and a switch S as shown in Fig. 210x.

(a) What current will flow?

(b) What will be the difference in potential between A and B, that is, between the terminals of the battery?

(c) Draw a diagram showing the different lettered points of the circuit arranged in order on a horizontal line beginning with A, and show the potential changes along the circuit by a graph which indicates potential by vertical distance above this horizontal line.

(d) Draw a similar diagram showing the potential changes along the circuit for the same circuit when the switch S is open.

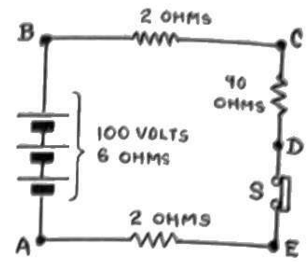


Fig. 210x

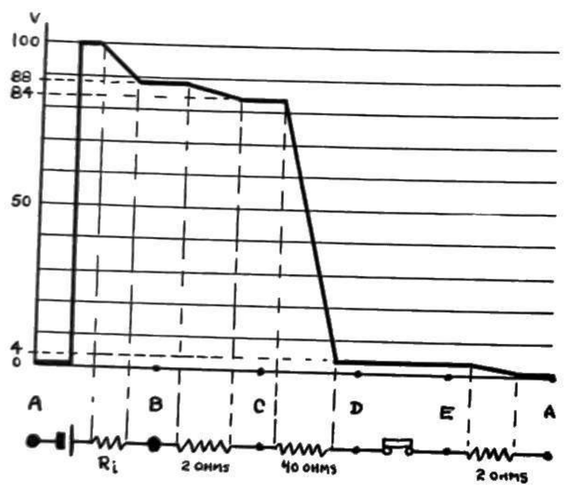
Solution: (a) From Eq. (161-2), page 49,

$$I = 100 \text{ volts}/(6 + 2 + 40 + 2) \text{ ohms}$$
$$= 2 \text{ amp}$$

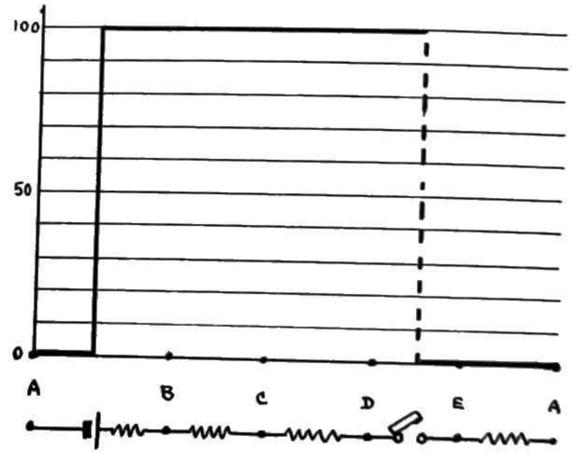
(b) From Eq. (156-2), page 48, the gain in potential V in passing from A to B through the battery is given by

$$V = 100 \text{ volts} - (2 \text{ amp} \times 6 \text{ ohms})$$
$$= 100 \text{ volts} - 12 \text{ volts}$$
$$= 88 \text{ volts}$$

(c)



(d)



210. A battery having E = 6 volts and an internal resistance R_i = 1 ohm is connected in a simple series circuit with a lamp having a resistance of 8 ohms and a rheostat R having a resistance of 3 ohms.
Find the answers to parts (a) (b) (c) and (d) of Prob. 210x, using the circuit shown in Fig. 210.

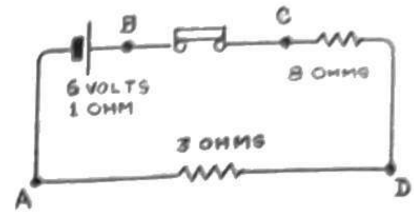
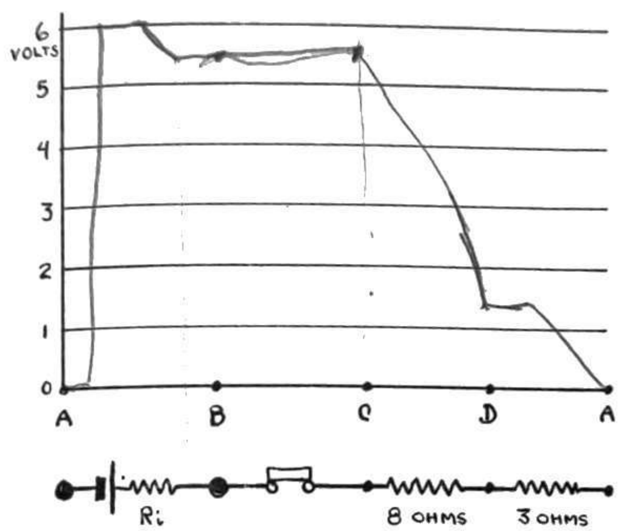
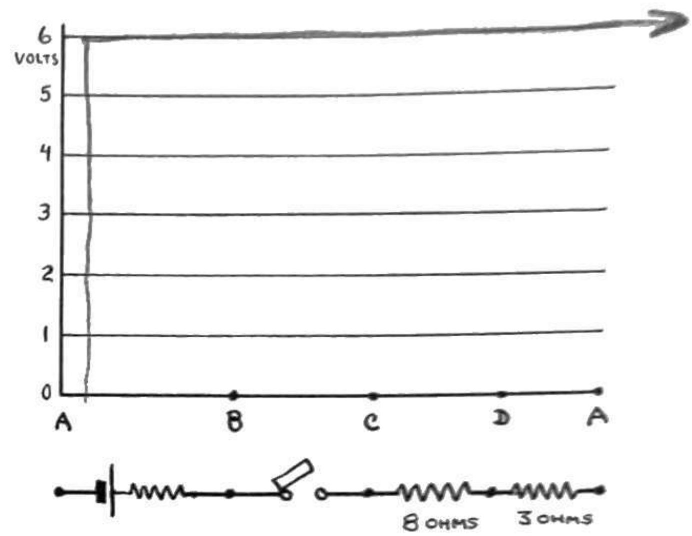


Fig. 210



Graph for Prob. 210(c)



Graph for Prob. 210(d)

211. A dry cell having an internal resistance of .05 ohm and an emf of 1.5 volts is to be used to furnish heat in a small resistance R of .05 ohm.
(a) What current will flow?
(b) What will be the difference in potential across R?
(c) How much energy will be liberated as heat in R in 30 seconds?
(d) How much energy will be wasted inside the cell as heat in the same time?

212. When a wire having a resistance of .4 ohm is connected between the terminals of a dry cell with an emf of 1.5 volts, the potential difference between the terminals falls to 1 volt. What is the current in the wire and what is the internal resistance of the cell?

213. A storage battery has an internal resistance of 0.03 ohms and an emf of 6 volts.
(a) What is the maximum current this battery could furnish?
(b) What would be the difference in potential between the terminals when furnishing this current?
(c) What becomes of the chemical energy expended?

214. A voltmeter taking negligible current is connected to an electrical outlet and reads 120 volts. When an electric heater marked 120 volts, 2880 watts (R = 5 ohms) is plugged in, the voltmeter reading drops to 100 volts. If it is known that the generator at the power station maintains a constant terminal voltage of 120 volts, what is the resistance of the line from the power house to the outlet?

214x. A generator having an emf of 40 volts and an internal resistance of .5 ohm is set up on a truck to furnish current to a tent 500 ft away. Wires having a resistance of .25 ohm for 500 ft of length are used for the connecting lines.
(a) What will be the potential difference across a lamp bulb taking 2 amperes if it is the only thing connected to the line at the tent?
(b) What will be the potential difference across the lamp if a stove is connected in parallel with the lamp such that the two together take 20 amperes?

Solution: (a) The generator, one side of the line, the lamp bulb, and the return side of the line are all connected in a series circuit. Using Eq. (161-1), page 49 gives

$$40 \text{ volts} = (2 \text{ amp} \times 5 \text{ ohm} + (2 \text{ amp} \times .25 \text{ ohm}) + (2 \text{ amp} \times R) + (2 \text{ amp} \times .25 \text{ ohm})$$

where R is the resistance of the lamp. Hence

$$40 \text{ volts} = (1 \text{ volt}) + (.5 \text{ volts}) + (X) + (.5 \text{ volt})$$

where X is the voltage across the lamp. Note that 1 volt is lost in the generator itself, and .5 volts in each side of the line. The energy involved in these losses will be dissipated as heat in the wires. Thus

$$X = (40 - 2) \text{ volts}$$
$$= 38 \text{ volts}$$

(b) Using the same method as in part (a) gives

$$40 \text{ volts} = (20 \text{ amp} \times .5 \text{ ohm}) + (20 \text{ amp} \times .25 \text{ ohm}) + (X) + (20 \text{ amp} \times .25 \text{ ohm})$$

or, $\quad 40 \text{ volts} = 10 \text{ volts} + 5 \text{ volts} + X + 5 \text{ volts}$

Hence $\quad X = (40 - 20) \text{ volts} = 20 \text{ volts}.$

215. (a) A voltmeter having a resistance of 100 ohms is connected to the terminals of a cell to find its emf, approximately. If the unknown emf is really 1.500 volts, and the internal resistance is 10 ohm, what will the voltmeter read?
(b) If a voltmeter having a resistance of 1000 ohms is used, what would the voltmeter read?

216. A voltmeter and a milliammeter are connected in series to a battery having an emf of 6 volts and negligible internal resistance. The voltmeter reads 5.95 volts and the milliammeter reads .05 amp.
(a) What is the resistance of each instrument?
(b) What current would flow if the voltmeter alone were connected to the battery terminals?
(c) What current would flow if the milliammeter alone were connected to the battery?

GROUP 22

220. What current will flow in the circuit shown in Fig. 220?

221. Find the emf and internal resistance of a single cell which could replace all three cells in the circuit of the preceding problem and give the same current.

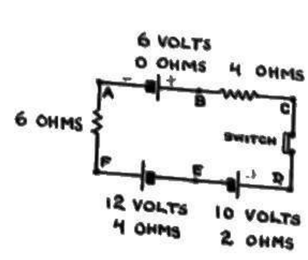


Fig. 220

GROUP 23

230. Two batteries as shown in Fig. 230 send a current of 6 amperes through R.
(a) Find the direction and magnitude of the current in the 2-volt battery.
(b) Compute the value of R.

230x. (a) Find the potential difference between A and B in Fig. 230x and state which point is higher.
(b) Find the direction and magnitude of the current in the 8-volt battery.
(c) Find the internal resistance of the 6-volt battery.

231. Four similar storage cells of negligible internal resistance and 2 volts emf are connected as shown in Fig. 231. What current will flow in the 10 ohm resistor?

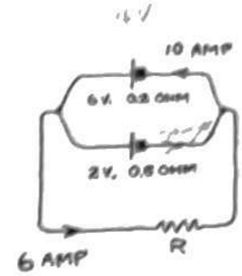


Fig. 230

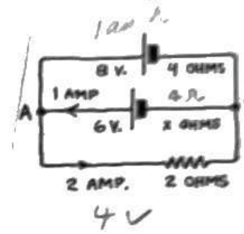


Fig. 230x

GROUP 24

240. In a slide-wire bridge, such as is illustrated in Fig. 240, the slide-wire AB has a length L of 100 cm. The distance AK is 40 cm, and R is 100 ohms. The emf of E is 3 volts and the cell has negligible internal resistance.
(a) What is the value of the resistance X?
(b) What current will be flowing through the resistance X when the bridge is balanced?
(c) If a battery E' of 6 volts were put in place of E, without changing anything else, would the bridge remain balanced?
(d) If a galvanometer having a different resistance were substituted for the original one, would the bridge still be balanced?

241. Two resistances, a galvanometer and a battery, are permanently connected as shown in Fig. 241. An unknown resistance, X, may be connected between either pair of binding posts, A or B, and a variable known resistance, R, may be connected between the other pair. If R may be made to have any value between 1 and 111 ohms, what is the smallest and what is the largest value of X that can be measured, using the arrangement as a Wheatstone's bridge?

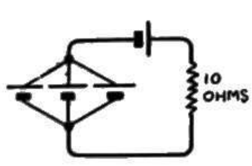


Fig. 231

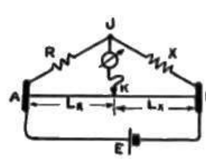


Fig. 240

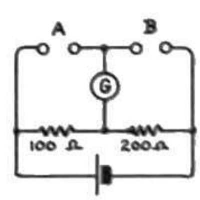
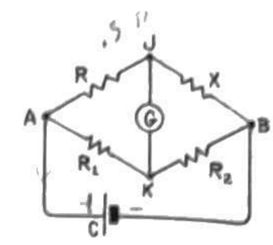


Fig. 241

242. In a bridge as shown in Fig. 242, R_2 = 8 ohms, R_1 = 4 ohms, the potential difference across R_1 is 5 volts and the current through R is 2 amp. Find the value of the resistance X and find the potential between A and B.



1,25·

Fig. 242

GROUP 25

250. A piece of copper wire has a resistance of 2 ohms. What would be the resistance of another piece of copper wire having twice the length and twice the diameter of the first piece?

251. What must be the diameter of a piece of nickel wire 4 meters long in order that it shall have a resistance of 2 ohms at 20° C?

251X. A tube of mercury 1 meter long has a resistance of 1 ohm at 20° C. What is the diameter of the tube?

253. A wire 20 ft long and .020 inches in diameter has a resistance of 5 ohms. What is its resistivity in ohm-C.M./ft?

253x. How many feet of nichrome wire .015 inches in diameter must be used in a 600 watt heater for use on 120 volts?

255. The resistance of a piece of steel wire having a diameter of .008 inch and a length of 20 feet is 25 ohms.
(a) What is the resistivity of this steel in ohm-C.M./ft?
(b) What would be the resistivity of this same steel in a wire having a diameter of .004 inches and a length of 40 ft?

256. (a) How would the area of an aluminum wire compare numerically with the area of a copper wire having the same resistance per foot of length?
(b) How would the diameter of the aluminum wire compare with that of the copper?

GROUP 26

260. The resistance of a wire at 20° C is 45.0 ohms. At 100° it is 55.0 ohms. What is its temperature coefficient of resistance?

261. If the temperature of a tungsten filament in a incandescent lamp is 1400° C when operating, what would be the resistance of a 120-watt, 120-volt lamp when the filament was at the temperature of melting ice, assuming a constant coefficient as given in the table on page 37.

GROUP 27

270. A battery B is connected to the ends of a uniform resistance wire W having a resistance of .02 ohms per cm. A cell C having an emf of 1.2 volts and galvanometer having a resistance of 100 ohms are connected together in series, and the two ends of this series combination are touched to two points of the wire W 60 cm apart. If the galvanometer shows no deflection, what current is flowing in the wire W?

271. In Fig. 271, AB is a uniform wire having a total resistance of four ohms and a length of one meter. S has an emf of 6 volts and a negligible internal resistance. C is a dry cell having an emf of 1.5 volts.
(a) What must be the resistance between T and B so that no current will flow through the galvanometer G?
(b) What would the length of wire between T and B have to be for balance?
(c) If the connections to the terminals of the cell C were reversed, leaving other connections the same, would it be possible to find a distance TB between A and B so that the current in G would be zero?

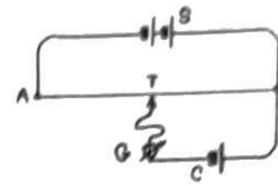


Fig. 271

272. A battery having an emf of 6 volts is used to furnish current in a potentiometer to balance the potential difference of a cell having an emf of about 1.2 volts. Show on a drawing how the cells should be connected. Mark the + and - terminals of the cells, the direction of the current in the potentiometer wire, the position of the galvanometer, and show approximately the position of the tap relative to the ends of the potentiometer wire.

273. A 2-volt battery of negligible internal resistance is used at B in Fig. 172 on page 52. The wire AD is 100 cm long and has a resistance of .04 ohm per cm of length. What must be the resistance W in order that a cell having an emf of .1 volt will be balanced by 10 cm of the slide-wire?

274. If a potentiometer circuit as shown in Fig. 172, page 52 is balanced, give the direction of the current, if any, that would flow through G if (a) T' is moved toward D, (b) W is increased, (c) the resistance of the galvanometer is increased by a rise in temperature of the galvanometer, (d) the emf of B is increased, (e) the internal resistance of C increases, (f) the internal resistance of B increases.

275. If the galvanometer in Fig. 275 reads zero when either key K_1 or K_2 is closed, what is the emf of C and what is the difference in potential across the 20 ohm resistor.

276. A cell C was connected to the ends of a uniform potentiometer wire 100.0 cm long, forming a closed circuit. When a standard cell (emf = 1.096 v) with a galvanometer was connected to two points on the wire 80.00 cm apart, it was found that the emf of the standard cell was balanced. Given that 10/8 of 1.096 is 1.37, is the emf of C equal to, larger than, or smaller than 1.37 volts? Explain.

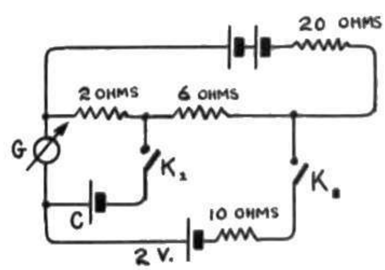


Fig. 275

277. If the galvanometer G in Fig. 277 reads zero, and the emf of C is 6 volts, what must be the value of the current I_1?

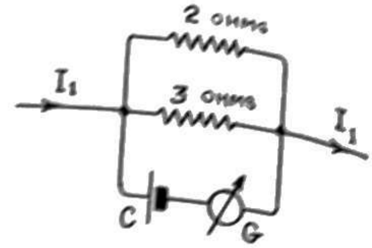


Fig. 277

GROUP 28

280. If a voltmeter having a resistance of 100 ohms reads 8 millivolts when connected to a thermocouple having a resistance of 20 ohms, what is the emf of the couple?

281. (a) Given $V = E - IR_c$ for the terminal voltage of a thermo couple in which E varies with the temperature. Prove that V as measured by a voltmeter is proportional to E. (b) A milli-voltmeter having a resistance of 8 ohms is used to read the terminal voltage V of a thermo couple having a resistance of 2 ohms and a variable emf E. If V = kE, compute the value of k.

GROUP 29

290. (a) The figure shows a graph of V vs I for a certain carbon arc A. Draw a graph on the same axes to show V vs I for a fixed resistance of 10 ohms.
(b) If the ten ohm resistance is connected in series with the arc A to a battery of negligible internal resistance, what emf would be required to give a current of 4 amperes?
(c) What resistance would be required in series with the arc to give a current of 7 amperes when the combination is connected to a 120 volt line?

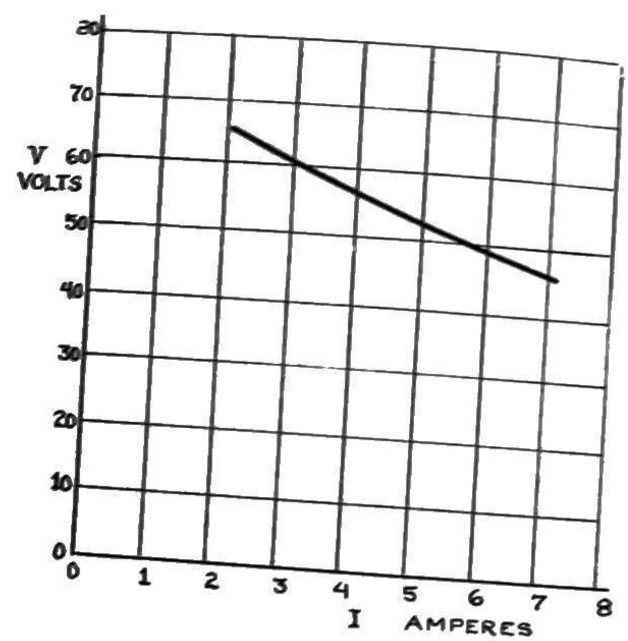


Fig. 290

GROUP 30

300. A horizontal wire 2 cm long carrying a current of 50 amperes to the east is forced to the south with a force of .5 gf. This force is the maximum force that can be found for any orientation of the wire at that point. What is the magnitude and direction of B at that point? Express in webers per m^2 and in gauss.

301. A straight horizontal wire runs east and west in the northern hemisphere in place where the earth's total field is 0.3 gauss and the angle of dip is 60^o. The wire carries a current of 20 amp to the west. What will be the magnitude and direction of the force on a 5 meter length of this wire?

302. What will be the magnitude and direction of the force on a 5 meter piece of wire placed horizontally in a north and south direction if the wire carries a current of 20 amp to the north in the northern hemisphere where the earth's total field is 0.3 gauss with an angle of dip of 60^o?

303. What will be the magnitude and direction of the force on the 5 meter piece of wire of the preceding problem if it is placed in a vertical position with the current flowing down?

GROUP 31

310. A coil 5 cm wide and 10 cm high is in a vertical north and south plane, where there is a uniform horizontal field of 0.4 gauss to the north. There are 100 turns in the coil and a current of 2 amperes is flowing down in the north side.
(a) What is the magnitude and direction of the force on the north side of the coil?
(b) What is the magnitude of the torque on the coil about a vertical axis?

GROUP 32

320. Positive charges shot into an evacuated space move around in horizontal circles in a clockwise direction as seen looking down. What is the direction of the field in that space?

321. (a) What will be the radius of curvature of an electron moving horizontally through a vertical field of 800 gauss if the electron has the velocity it gained in moving through a difference in potential of 2000 volts. Assume the mass of the electron is as given in Sect. 12 for an electron at rest.
(b) If the field is directed vertically downward, will the electron move clockwise or counterclockwise as viewed from above?

GROUP 33

330. At a point P on the equator, the earth's field B_e is .30 gauss. A magnet NS is placed horizontally on a line extending east of P. The field at P due to the magnet NS is a horizontal field of .40 gauss.
(a) What is the magnitude and direction of the total field at P?
(b) In what direction would the north pole of a compass point if placed at P?

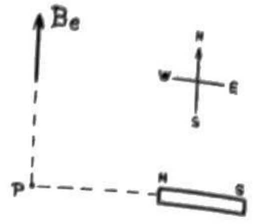


Fig. 330

331. At the same point P as in the preceding problem, a magnetic compass needle points its N pole 30^0 to the east of north when a magnet AB is placed as shown in Fig. 331.
(a) What is the field of the magnet AB at P?
(b) Is the end A a north or a south pole?

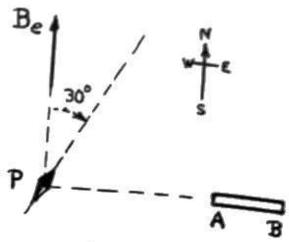


Fig. 331

332. Fig. 332 shows a bar magnet AB lying on a horizontal table top as viewed from above.
(a) If the resultant horizontal field at P is zero due to both the earth's field and that of the magnet AB, which pole is at the A end of the magnet?
(b) If the resultant horizontal field at Q is zero, which pole is at the A end of the magnet?

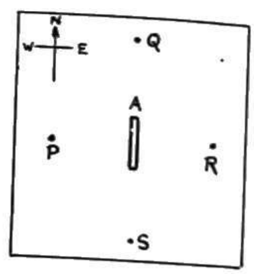


Fig. 332

333. Fig. 333 shows a horizontal table top as viewed from above. State in which of the following positions a bar magnet could be laid on the table to make a compass needle at 0 deflect away from the northward direction to the east.
(1) at R with the N pole to the west.
(2) at S with the N pole to the west.

334. The value of the horizontal component of the earth's field at a certain point is .21 gauss. What is the vertical component of the field if the angle of dip is 60^0?

335. The horizontal component of the earth's field is .3 gauss and the angle of dip is 30^0 at a certain place in the northern hemisphere. What will be the magnitude and direction of the force on a horizontal piece of wire 15 meters long running north and south carrying a current of 200 amperes to the south?

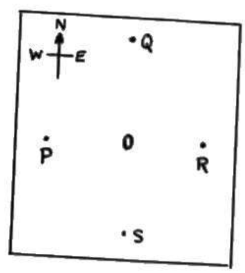


Fig. 333

GROUP 34

340. The flux density of the earth's field at the north (artic) magnetic pole is .25 gauss.
(a) How many lines of flux representing webers will pass through a horizontal area of 1 square meter?
(b) How much horizontal area in meters is allocated to each line of flux in webers?
(c) How many lines of flux representing microwebers will pass through a horizontal area of 1 square meter?

341. The lines of force representing the field of a cyclotron magnet emerge perpendicularly from the surface of the north pole of the magnet. The face of the pole is a round flat surface having a diameter of 1 meter. The average flux density in the space near this surface is .5 web/m^2.
(a) How many lines of force leaving the pole face would be needed to represent the flux from the face of the pole measured in webers?
(b) How many lines would be needed to represent the flux in microwebers?
(c) How much force would this field exert on a straight wire carrying a current of 30 amperes parallel to the face of the pole, assuming the field is negligible beyond the edge of the pole face.

342. The flux density of the earth's field is 2×10^{-5} webers per square meter at a point in the northern hemisphere where the angle of dip is 30^0.
(a) What will be the flux in webers through a horizontal area of 1 square mile? (Use 1 square mile = 2.59×10^6 square meters).
(b) How many lines representing the flux in microwebs will pass through a window in a vertical east and west wall if the window is 1 meter wide and 2 meters high?

GROUP 35

350. (a) A horizontal trolley wire OAB is 5 meters directly above a point P on the ground as shown in Fig. 350. If AB is a 2 mm section of the wire, and r is 8 m, what will be the magnitude and direction of the magnetic field at P due to the little section AB if the current in the wire is 200 amperes flowing northward?
(b) If the distance OB is 3100 times as great as the distance AB, will the field at P due to OB be 3100 times that due to AB? Explain.

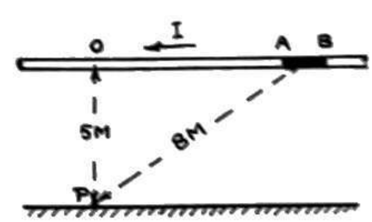


Fig. 350

351. A current is flowing east in a straight level section of a wire shown in Fig. 351. The points P, R, and T are each 10 cm from a point 0 on the wire, and all four points lie in the same vertical plane. The section AB is 0.02 cm long, and a current of 1500 amperes is flowing as indicated by the arrow. What will be the direction and magnitude of the magnetic field due to the current in the segment AB at each point P, R, and T?

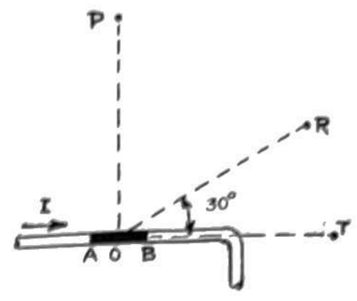


Fig. 351

352. (a) In Fig. 352 the length AB is 6 cm, BC is 7.85 cm along a circular arc 5 cm in radius, and CD is 6 cm. A current of 15 amperes is flowing through ABCD. What will be the field at the center 0 of the arc due to the currents in the conductor between A and D?
(b) If the wire CD carrying the current away is moved through an angle θ, as shown, to a position CE, will the field at 0 be increased, decreased, or unaffected? Explain.

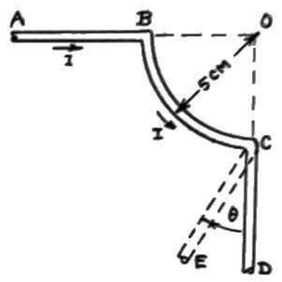


Fig. 352

GROUP 36

360. Compute the flux density at the center of a flat circular coil of 60 turns having a radius of .12 m due to a current of 15 amperes.

361. A flat circular coil having three turns with a radius of 5 m is lying on a horizontal table top. If a current of 250 amperes is flowing clockwise in the coil as viewed from above, what is the magnitude and direction of the magnetic field produced at the center of the coil?

362. An arc of wire 20 meters long having a radius of curvature of 15 meters is lying on a floor as shown in Fig. 362. A current of 3000 amperes is flowing in the direction indicated in the figure. What is the magnitude and direction of the field produced at the center of curvature C due to the 20 meters of wire?

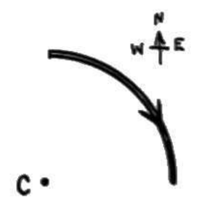


Fig. 362

GROUP 37

370. A tangent galvanometer(1) has a coil of 12 turns with a radius of 15 cm. The horizontal component of the earth's field is .2 gauss. The north pole of the compass needle is deflected 60° to the east by a current in the coil.
(a) What is the field at the center of the coil due to the current?
(b) What is the current in amperes?
(c) Is the current flowing up or down in the north side of the coil?

371. (a) What current in amperes will be required to give a deflection of 45° in a tangent galvanometer with 1500 turns having a radius of 20 cm, if the horizontal component of the earth's field is .2 gauss?
(b) What would be the field at the center of the coil due to this current?

372. Describe the position required for the coil in a tangent galvanometer, and the direction of current flow required to deflect the north pole of the needle to the east.

373. A tangent galvanometer is set by mistake with the coil in an east and west vertical plane. A current I flows through the coil in a clockwise direction as seen looking to the south, and is steadily increased from a value of zero to a value that would produce a 60° deflection if the galvanometer were properly used. What change, if any, would occur in the deflection of the needle as the current is increased?

(1) A flat coil of wire placed in a vertical north and south plane with a compass needle at the center of the coil constitutes a tangent galvanometer. A horizontal cross-section through such an arrangement is shown in Fig. 370 as it would appear looking downward. If the current is flowing up in the south side of the coil, as indicated, the field B_c due to the coil will be a westward field, as shown. The magnitude of B_c will be given by

$$B_c = \mu_0 NI/2R \qquad (1)$$

The compass needle at the center of the coil will take up the position of the resultant of B_c and the horizontal component B_e of the earth's field. If θ is the angle by which the compass needle is deflected away from the north, the tangent of θ will be B_c/B_e. Thus,

$$B_c = B_e \tan\theta. \qquad (2)$$

Substitution in equation (2) from equation (1) gives

$$\mu_0 NI/2R = B_e \tan\theta \qquad (3)$$

or,

$$I = \frac{2B_eR}{\mu_0 N}\tan\theta \qquad (4)$$

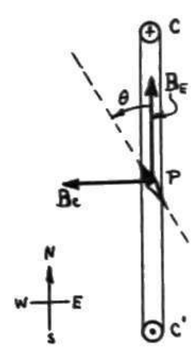


Fig. 370

Thus, if the earth's field is known, the current can be computed from the observed deflection, and the arrangement can be used as a galvanometer to measure current. The tangent galvanometer is seldom used in practice, but the principles involved in its operation are of fundamental importance.

GROUP 39

390. A straight trolley wire five meters above the street extends far in either direction. If it carries a current of 200 amperes to the east, what will be the direction and magnitude of the field due to this current at a point on the street directly below the wire?

391. (a) How many amperes of current must flow upward in a long vertical wire to give a field of 1000 microweb/m^2 at a point P, 8 cm east of the wire, assuming that the wire extends far above and below this point, and what will be the direction of the field at P?

GROUP 40

400. A current of 5 amp flows in a large solenoid 1.5 meters in diameter and 20 meters long which is uniformly wound with 8000 turns. What is the flux density and the flux through the solenoid at a point far from either end? (Cross-sectional area $= \pi R^2 = 1.77\ m^2$).

401. A current of 12 amperes flows in a uniformly wound solenoid which is 1 m long and has a total of 1600 turns. The cross-sectional area is 4 cm^2, or .0004 m^2.
(a) Compute the flux density at the center of the solenoid.
(b) Compute the total number of lines of flux through the solenoid at its center to represent the flux in webers and in microwebers.

402. A long thin solenoid has 1200 turns for each 80 cm of length, and has a cross-sectional area of 2 cm^2.
(a) What current in amperes will be required to produce a field of .006 web/m^2 in the solenoid?
(b) What will be the flux through the solenoid at its center?

403. What current would have to flow in a long solenoid having 1200 turns per m and an area of 5 cm^2, if we wish to have a magnetic flux of 3 microwebs through the solenoid at the middle?

GROUP 45

451. Two parallel wires 3 cm apart extend north and south. One is directly above the other. The top wire carries a current of 15 amperes north and the lower wire carries a current of 45 amperes in the same direction.
(a) What is the flux density at the lower wire due to the current in the upper wire?
(b) What is the magnitude and direction of the force on each meter of the lower wire?

452. For the two wires of problem 451, compute the flux density at the top wire due to the current in the lower wire, and then compute the force exerted by this flux density on one meter of the upper wire.

GROUP 49

490. A permanent magnet having a flux of 20 microwebs is inserted in a coil of 300 turns in a time of .2 sec. What will be the average emf induced during the .2 second interval?

491. A solenoid having 1000 turns per m of length is surrounded at its center by a flux measuring coil C of 3 turns. The area of the solenoid is 8 cm^2 and the coil C fits so closely that its area may be taken to be the same. The current in the solenoid is increased uniformly from 2 amperes to 5 amperes in 2 seconds.
(a) What is the increase in the flux through the coil C?
(b) What is the emf induced in the coil C while the current is increasing.

GROUP 50

500. (a) A straight piece of wire 10 cm long forms a part of the armature winding in a large generator. This piece of wire is perpendicular to a magnetic field of 1 weber/m^2, and as the armature rotates, the wire is carried 50 cm in .2 seconds in a direction perpendicular to both the field and wire. What is the emf induced in this piece of wire?
(b) How much force will be required to keep this piece of wire moving if a current of 200 amperes flows in the wire?
(c) What will be the emf and force for the 10 cm piece of wire if no current flows, the field and speed of rotation remaining unchanged.

GROUP 51

510. A flux meter consists of a 200 turn coil having a resistance of 5 ohms connected in series with a ballistic galvanometer having a resistance of 80 ohms. If it requires 2 microcoulombs of charge to deflect the galvanometer one cm, how much change in flux through the coil will deflect the galvanometer 10 cm?

511. A ballistic galvanometer is connected to a coil C to serve as a fluxmeter as shown in Fig. 511. The fluxmeter is to be calibrated by using a solenoid AB to furnish a known change in flux in the coil C. When a field of .0040 web/m^2 is established in the solenoid by turning on the current in the solenoid, the ballistic deflection of the galvanometer is 4 cm. If the solenoid has a cross-sectional area of 2 cm^2, what is the calibration of the fluxmeter in lines of flux per cm of deflection?

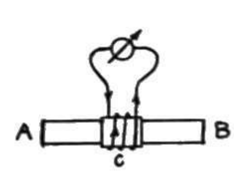


Fig. 511

512. A rectangular coil 50 cm x 80 cm having 500 turns is connected to a ballistic galvanometer to form a fluxmeter. The coil and galvanometer together have a resistance of 200 ohms. If the coil is quickly rotated from a position parallel to the earth's field to a position perpendicular to the field, a charge of 20 microcoul passes through the galvanometer. What is the value of the earth's field at that point, in gauss?

513. How much charge will pass through the galvanometer of Problem 512 if the coil is rotated 180° from one position perpendicular to the field to another perpendicular position?

514. How much charge will pass through the galvanometer of Problem 512 if the angle of dip is 30° and if the coil is rotated about a vertical axis from a vertical north and south plane to a vertical east and west plant?

515. A permanent magnet having a cross-sectional area of 2 cm² is inserted as shown into a coil of 80 turns. The coil and a galvanometer in series have a resistance of 4 ohms, and a charge of .0002 coulomb flows when the magnet is inserted.
(a) What is the total flux at the center of the magnet?
(b) What is the flux density at the center of the magnet?

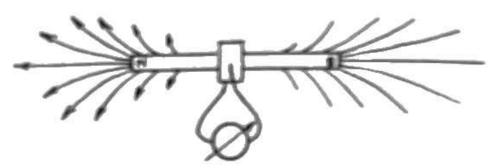


Fig. 515

GROUP 53

In all problems below that involve the magnetic properties of materials it is to be assumed that the material is in the form of an effectively endless bar magnetized parallel to its length, and that it is in an effectively endless solenoid, unless otherwise specified.

530. An unmagnetized bar having a cross section of 4 cm² is in a solenoid of the same size. A current that produces a magnetizing field B_O of 500 microweb/m² in the solenoid is turned on. A galvanometer connected to a flux measuring coil around the solenoid is deflected 10 cm, indicating an increase in flux of 24 microwebs.
(a) What is the total flux density B in the bar while the current is flowing?
(b) What is the relative permeability of the bar for this field?
(c) When the current is turned off, a ballistic deflection of 4 cm results in the galvanometer, in the opposite direction to the deflection which occurred when the current was turned on. What flux density B_R remains in the bar as permanent magnetization?

531. The entire flux through a solenoid with a core is 100 microweb. The area of the solenoid which is completely filled with the core is 10 cm², or ~~.001 m²~~. The field due to the current in the solenoid is 6000 microweb/m².
(a) What is the flux density in the solenoid when the core is in place?
(b) What is the relative permeability of the core under these conditions?
(c) If the current in the solenoid were kept constant and the core were withdrawn, how much flux would be left in the solenoid?

532. What will be the total flux through a core if the flux density of the magnetizing field is 2000 microweb/m², the cross-sectional area of the core 75 cm², and the relative permeability for that field is 800?

533. If the magnetizing field in the preceding problem were increased by any ratio, would the number of lines of flux be increased in the same ratio? Explain.

534. A solenoid having an area of 5 cm^2 is completely filled with a permalloy core. The total flux is 500 microweb when the flux density produced by the solenoid current alone is 1000 microweb/m^2.
(a) What is the relative permeability of the permalloy under those conditions?
(b) If the permalloy is magnetically saturated in part (a), what will be the entire flux in the solenoid if the magnetizing field is increased by a factor of 20?

GROUP 54

540. A short permanent bar magnet 8 cm from pole to pole is placed crosswise inside a long solenoid at a point far from either end. The solenoid has 6000 turns in 5 meters of length with a current of 15 amp. If it required a torque of 117 gf cm to hold the magnet perpendicular to the field in the solenoid, what is the pole strength of the magnet?

541. A horizontal magnet suspended to swing freely about a vertical axis in the earth's field oscillates with a period of 12 seconds. A permanent magnet is placed on a table near by and the first magnet then oscillates with a period of 3 seconds. If the horizontal component of the earth's field is .2 gauss, what is the horizontal component of the total field existing after the second magnet was placed nearby?

GROUP 56

560. (a) Express the efficiency of a generator in terms of the input torque L, the angular velocity ω, the output voltage V, and the output current I.
(b) Starting with the mechanical input power L ω, draw a diagram showing how this power is divided between the various losses and the useful output (VI) of electrical power. Consider eddy current and hysteresis losses as lumped in with the mechanical losses. (Solution given).

Fig. 560

561. Starting with the electrical input power (= VI) for a motor, draw a diagram similar to the above, showing the power losses and energy transformations through to the mechanical power output, and express the efficiency in terms of L, ω, I and V.

562. A series wound motor takes a current of 0.5 amp from a 110 volt line. The field resistance is 8 ohms and the armature resistance is 12 ohms. How much electrical power is converted into mechanical power (including mechanical losses)?

563. The motor of problem 562 is disconnected from the 110 volt line and is connected to a resistance such that a current of .5 amp flows when the machine is turned at the same speed by some source of mechanical power to serve as a generator. What will be the output voltage?

564. A shunt wound generator has an output voltage of 120 volts when furnishing a current of 30 amp. The field resistance is 60 ohms, and the armature resistance is .5 ohm.
(a) What is the induced emf?
(b) What is the resistance of the load connected to the generator?
(c) How much power will be required to keep the generator running if the mechanical losses amount to 250 watts?
(d) What is the efficiency?
(e) If the electrical resistance of the load be decreased, while the speed of the generator is kept constant, what will be the effect on the induced emf, on the mechanical power required to keep the generator running, and on the output voltage?(1)

565. A shunt motor has a fixed input voltage of 110 V. The armature current is 5 amp. The armature has a resistance of 2 ohms, and the field has a resistance of 55 ohms.
(a) What is the electrical power used?
(b) What is the back emf?
(c) If the mechanical load is increased so that the speed is reduced 2%, what is the per cent change in the back emf? In the armature current? In the amount of electrical power converted into mechanical power? State whether it is an increase or a decrease in each case.(1)

566. The armature resistance of an automobile generator G in Fig. 566 is 0.5 ohms. The storage battery S has a constant emf of 6 volts and a negligible internal resistance. The lamps L have a combined resistance of 1 ohm. The current taken by the field winding is negligible, and that winding is not represented in the figure. The induced emf in the generator is 8 volts.
(a) Find the direction and magnitude of the current in G.
(b) Find the direction and magnitude of the current in L.
(c) Find the direction and magnitude of the current in S.

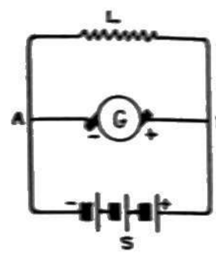


Fig. 566

567. Repeat Problem 566 for an armature speed such that the induced emf is 12 volts.

568. Repeat Problem 566 where the induced emf is 4 volts.

569. Find the value of the emf that would have to be generated so that no current would flow in the battery, in the circuit of Problem 566.

GROUP 58

580. What would be the time required for a 10 mfd condenser to discharge to 1/2.72 of its original voltage if it is discharged through a resistance of (a) 5000 ohms, (b) 2 megohms?

(1) In problems 564 and 565 assume that the field remains constant. This would be approximately the case if the field magnet was saturated.

581. Show that 1 ohm farad is equal to 1 second.

582. What is the capacitance of a condenser if it discharges to 1/2.72 of its original value in .08 seconds through a resistance of 50,000 ohms.

583. A condenser having a capacitance of 2 mfd is discharged through a resistance R which gives the curve A of Fig. 321, Sect. 321. (a) What is the value of R? Find the current at the instant the condenser is half discharged. (The slope of the curve may be determined graphically when needed).

583x. Solve problem 583 for a resistance which gives the curve B.

GROUP 59

590. A 1 mfd condenser connected in a circuit as shown in Fig. 590 is charged by moving K over to F, then discharged by moving K over to E. If this is repeated 50 times per second, what is the average current flowing through the galvanometer G? The potential difference of the cell terminals is 3 volts.

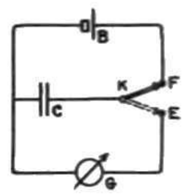


Fig. 590

591. If the condenser of the preceding problem had a capacitance of 8 microfarads, and was charged and discharged by moving K back and forth 120 times per second, what would be the magnitude and direction of the average current through the galvanometer?

592. Referring to Fig. 590, a galvanometer reads 40 microamperes when the difference in potential furnished by B is 4 volts and the key is operated 20 times per sec.,
(a) What is the value of C?
(b) What would the galvanometer read if placed in the line between B and F, and would the current flow from B to F, or vice versa, when the key was operated?
(c) What would be the galvanometer reading if it were placed between C and K?

GROUP 60

600. An emf of 8 volts is induced in a secondary coil when the current in the primary changes at the rate of 400 amp per sec. What is the mutual inductance between the two coils?

601. Two coils have a mutual inductance of 60 mh. A current in the primary increases uniformly from 0 to 2 amp in .001 sec. What will be the emf in the secondary coil?

602. Can you describe an arrangement whereby two large coils might be close together and still have zero mutual inductance?

603. Explain why the mutual inductance between two coils may not be constant for all values of the current in the first coil if an iron core is used in either coil.

GROUP 61

610. An emf of 20 volts is induced in a coil when a current increases at the rate of 500 amperes per second. What is the self-induction of the coil expressed in henries and in millihenries?

611. How much energy is stored in the magnetic field of a 4 henry coil when it carries a current of 50 amperes? Show how the unit for the answer follows from the data.

GROUP 62

620. Fig. 620 shows the rate of growth of current in two different coils when each one is connected to a 12 volt battery of negligible internal resistance. The dotted lines represent the initial slopes of the graphs. Deduce the approximate value of the inductance and resistance of each coil.

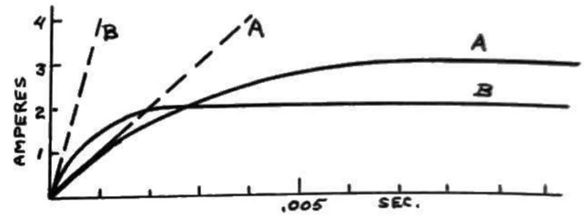


Fig. 620

621. A cell of 6 volts is connected in series to a coil of wire having a resistance of 2 ohms and an inductance of 0.5 henry.
(a) What will be the rate of increase of the current at the instant the switch is closed?
(b) What will be the final value of the current?
(c) What will be the rate of increase of the current when the current has reached one-fourth of its final value?

622. A circuit containing a battery of 12 volts is sending a current of 10 amperes through a coil connected in series with it. If the circuit is broken, can the emf across the break be greater than 12 volts? Can the current in the inductance when the spark occurs be greater than 10 amperes? Explain.

GROUP 75

750. A flat generator coil is rotating 20 times per second in a field such that the maximum value of the emf is 100 volts. The phase angle of the generated emf is zero when t is zero.
(a) Find the instantaneous emf in this coil when the phase angle is 0°, 30°, 90°, 150°, 180°, 270°.
(b) What will be the algebraic sign of the generated emf for phase angles in each of the four quadrants of a complete cycle?
(c) What will be the instantaneous value of the emf at a time 1 and 7/12 periods after $t = 0$?

751. (a) A coil mounted on a shaft is rotating in a uniform magnetic field. The maximum value of the emf is 100 volts, and the coil makes 20 revolutions per second. Draw a graph of the instantaneous emf e_1 plotted against time for three complete revolutions. Start counting time when e_1 is zero and increasing. Let 4 cm on your graph represent 1/20 sec.
(b) Draw a second curve labeled e_2, on the same axes used in part (a), showing the emf induced in a second similar coil mounted on the same shaft so that it is always 60^o behind the first in rotation. The maximum emf is the same in both coils.
(c) Show the reference vectors for the two emfs at a time = 0, on the same figure.
(d) Express the instantaneous value of e_2 in terms of the maximum emf, the angular velocity ω, the time t, and constants as needed.
(e) What is the value of ω in rad/sec?
(f) What are the instantaneous) values of e_1 and e_2 when t = 4/3 periods?

GROUP 76

760. The power expended as heat in a resistance of 20 ohms by a sinusoidal a.c. current of 60 cycles per second is 320 watts.
(a) What is the effective current, and the maximum current?
(b) What is the maximum difference in potential between the ends, and what is the effective difference in potential?

761. What is the maximum difference in potential between the wires of a 110 volt a.c. line?

762. During what per cent of the total time does the magnitude of the instantaneous current exceed the effective value for an alternating current?

GROUP 78

780. What must be the value of an inductance L if a maximum emf of 8 volts at 60 cycles per second sends a current with a maximum value of 6 amps through L?

781. Check the answer to the above problem by a different method using Fig. 347 in Sect. 347 as follows: Mark a time scale in seconds on the time axis for a 60 cycle current, and determine by graphical measurement the maximum value of di/dt, which occurs at the point F. Using the value of L obtained in the preceding problem with this maximum value of di/dt, compute the maximum emf, which should be 8 volts.

782. A sinusoidal emf having a rms value of 150 volts is applied in a path having an inductance of .2 henries.
(a) If the frequency is 60 cycles per second, how much current will flow?
(b) If the frequency is 1000 cycles per second, how much current will flow?

GROUP 79

790. A 32 mfd condenser is connected across an outlet of a 113 volt line. What current will flow in the path (a) if the frequency is 60 cycles per second? (b) if the frequency is 6000 cycles per second?

791. The maximum current in Fig. 348-2 of Sect. 348 is 3 amp, and the maximum emf is 80 volts. If the frequency is 60 cycles per second, what is the capacitance to which these curves apply?

792. Check the answers of the preceding problem by a different method, as follows: Lay off a time scale in seconds on the time axis of Fig. 348-2, and compute from graphical measurements the maximum value of de/dt. Then use this maximum value of de/dt with the value of C as found in the preceding problem to compute the maximum current which should be 3 amp.

795. A condenser allows a current of .05 amp to flow when connected to a 50 cycle alternating voltage of 110 volts. (a) What is the impedance?
(b) What would be the impedance at 10,000 cycles per second?

796. A resistance of 2 ohms, an inductance of .001 henry and a capacitance of 5 mfd are listed in the table below. Compute the impedance for each one at each of the frequencies given and fill the values in the table.

	ω	Frequency	Resistance	Inductance	Capacitance
(a)	0	(d.c.)			
(b)	500	(About 80 per sec.)			
(c)	6,000,000	(About 1000 kilocycles)			

(1000 kilocycles is in the range of Radio Broadcasting frequencies)

797. Compute the current that would flow through each of the paths of the preceding problem if an emf of 10 volts was applied to each.

	ω	Resistance	Inductance	Capacitance
(a)	0			
(b)	500			
(c)	6,000,000			

GROUP 80

800. The curves of Fig. 800 represent the respective emfs in two paths connected in series. The equations for these emfs are

$$e_1 = (6v) \sin \omega t$$

$$e_2 = (8v) \sin (\omega t + 90^o)$$

(a) On this figure, add the ordinates of the two curves at intervals of an eighth of a period and draw in the curve for the emf e applied to the combination.
(b) If $e = E_0 \sin (\omega t + \alpha)$, deduce from your curve the values of E_0 and α by graphical measurements.
(c) Record the values of e_1, e_2, and e at a time t = 3/8 period as determined from the curves.

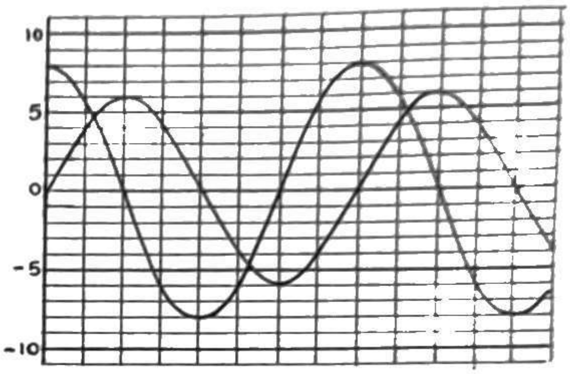


Fig.800

801. Add the emfs e_1 and e_2 given in the preceding problem by adding the rotating vectors for these quantities, and compare the values of E_0 and α found with those found there.

802. Two emfs, $e_1 = (8v) \sin \omega t$ and $e_2 = (16v) \sin(\omega t + 240^o)$ are connected in series.
(a) Show the positions of the rotating reference vectors at zero time, drawn to scale.
(b) On the same figure, show the rotating vector for the resultant emf e.
(c) If the resultant emf is written in the form $e = E_0 \sin (\omega t + \alpha)$, give values of E_0 and α.

803. Two generator coils rotate together. The effective emf in each is 120 volts. The first coil has zero phase angle when t = 0, and the second coil is 60^o behind the first. If the two coils connected in series give a resultant emf $e = E_0 \sin (\omega t + \alpha)$, what will be the value of α and what will be the effective value of the resultant emf?

GROUP 81

810. A 60 cycle sinusoidal current having a maximum value of 5 amp with a phase angle of zero flows through a resistance of 6 ohms and an inductance of .02 henry in series. Compute the values of $(E_0)_L$, E_0, and α, using the notation of Sect. 361.

811. What would be the readings E_R, E_L, and E of voltmeters reading effective values when these meters are connected respectively across R, L, and the series combination of R and L as given in problem 810?

812. (a) Plot the current of problem 810 against time with 1 cm = 5 amp and 4 cm = 1/60 sec, and t = 0 when the current is zero and increasing.
(b) On the same axis plot with a dotted line (.....) the emf e_R across the resistance, and with a dashed line (-----) the emf e_L across the inductance. Show the position of the reference vectors for e_R and e_L on a reference circle at the time t = 0. Let 1 cm = 20 volts. On the same figure, show also the reference vector for the emf e across the combination of R and L.

GROUP 83

830. (a) A resistance of 3 ohms is connected in series with a pure inductance that has an impedance of 4 ohms at 60 cycles. What current will flow when 110 volts is applied to the combination?
(b) What is the magnitude of the inductance?
(c) What current would flow if a 110-volt emf having a frequency of 1000 cycles per second were applied to the two in series?
(d) How much current would flow if a constant emf of 110 volts were applied?

831. When a 60 cycle emf of 100 volts is applied to an inductor, a current of 2 ampere flows. When a d.c. emf of 100 volts is applied, a current of 2.5 amperes flows. What is the impedance of the inductor at 60 cycles, and what is its inductance?

832. What is the impedance of a series combination of a resistance of 10 ohms and an inductance of .01 henry for a frequency of 60 cycles?

GROUP 84

840. An inductance L of .01 henry, a resistance R of 5 ohms, and a capacitance C of 200 mfd are connected in series. A 60-cycle alternating current having a maximum value of two amperes with a phase angle of zero flows through the combination.
(a) Compute the impedance of each part of the path separately, compute the impedance of the combination, and show the relation between these on a vector impedance diagram.
(b) Find the maximum emf across each part of the path and show all these on a reference circle diagram at a time when the current is zero and increasing.
(c) If the total emf applied in the combination is $e = E_0 \sin(\omega t + \alpha)$, find E_0 and α and show the rotating vector for e on the diagram of part (b).

841. A resistance of 3 ohms, an inductance of .204 henrys and a capacitance of 5 mfd are connected in series to an emf of 10 volts, having a frequency such that $2\pi f = 1000$ per second. Compute (a) the impedance of the condenser; (b) the impedance of the series combination; (c) the current through the condenser; (d) the voltage across the condenser.

842. A 2 μfd condenser can stand a difference in potential of 450 volts. Would it be possible to injure this condenser with too much voltage by connecting it in series with an inductor to a 110-volt, 60-cycle line?

843. A resistance, an inductance, and a condenser are connected in series. The effective value of the emf across the inductance is 50 volts, that across the resistance is 80 volts and that across the capacitance is 30 volts. The frequency is 60 cycles per second. The resistance is 20 ohms.
(a) What is the effective emf across the combination?
(b) What is the impedance of the combination?

GROUP 87

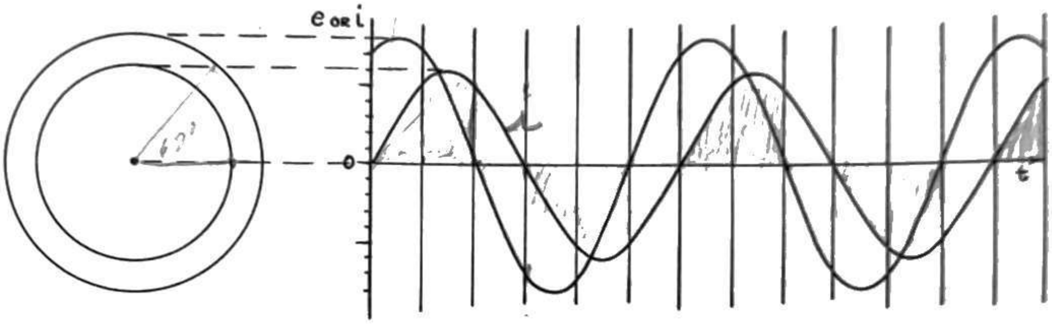


Fig. 870

870. The curves of Fig. 870 show an emf $e = (80v)\sin(\omega t + 60^{\circ})$ which is required to send a current $i = (6\ amp)\sin \omega t$ through a coil.
(a) In the reference circles indicated, draw the vectors for e and i at t = 0.
(b) Indicate on the graph the fractional parts of each period during which power is being expended by the emf.
(c) What is the average power expended by the emf?

GROUP 88

880. A toy train transformer furnishes 10 amp at 6 volts to operate the train. The secondary coil has fifteen turns and the primary voltage is 120 volts.
(a) Compute the current taken from the 120 volt power outlet assuming that the transformer acts as an ideal transformer.
(b) How many turns must there be on the primary coil of this transformer?

881. It is desired to construct a step-up transformer to furnish 40 milliamperes of current to a neon sign at 9,000 volts, with a primary coil having 100 turns of wire connected to a 120 volt power supply.
(a) How many turns must be used in the secondary coil?
(b) What current will the transformer take from the 100 volt line?

882. A power line between two cities has a resistance of 20 ohms.
(a) Compute the power lost in this resistance when 6,000 kilowatts of power are transmitted to give 20,000 volts with a power factor of 1 at the end of the line.
(b) Compute the power lost in the same line if the same power is transmitted to give 120 volts at the end.

Table of Trigonometric Functions.

°	Sin	Tan	Cot	Cos	°
0	0.0000	0.0000	∞	1.0000	90
1	0.0175	0.0175	57.2900	0.9998	89
2	0.0349	0.0349	28.6363	0.9994	88
3	0.0523	0.0524	19.0811	0.9986	87
4	0.0698	0.0699	14.3007	0.9976	86
5	0.0872	0.0875	11.4301	0.9962	85
6	0.1045	0.1051	9.5144	0.9945	84
7	0.1219	0.1228	8.1443	0.9925	83
8	0.1392	0.1405	7.1154	0.9903	82
9	0.1564	0.1584	6.3138	0.9877	81
10	0.1736	0.1763	5.6713	0.9848	80
11	0.1908	0.1944	5.1446	0.9816	79
12	0.2079	0.2126	4.7046	0.9781	78
13	0.2250	0.2309	4.3315	0.9744	77
14	0.2419	0.2493	4.0108	0.9703	76
15	0.2588	0.2679	3.7321	0.9659	75
16	0.2756	0.2867	3.4874	0.9613	74
17	0.2924	0.3057	3.2709	0.9563	73
18	0.3090	0.3249	3.0777	0.9511	72
19	0.3256	0.3443	2.9042	0.9455	71
20	0.3420	0.3640	2.7475	0.9397	70
21	0.3584	0.3839	2.6051	0.9336	69
22	0.3746	0.4040	2.4751	0.9272	68
23	0.3907	0.4245	2.3559	0.9205	67
24	0.4067	0.4452	2.2460	0.9135	66
25	0.4226	0.4663	2.1445	0.9063	65
26	0.4384	0.4877	2.0503	0.8988	64
27	0.4540	0.5095	1.9626	0.8910	63
28	0.4695	0.5317	1.8807	0.8829	62
29	0.4848	0.5543	1.8040	0.8746	61
30	0.5000	0.5774	1.7321	0.8660	60
31	0.5150	0.6009	1.6643	0.8572	59
32	0.5299	0.6249	1.6003	0.8480	58
33	0.5446	0.6494	1.5399	0.8387	57
34	0.5592	0.6745	1.4826	0.8290	56
35	0.5736	0.7002	1.4281	0.8192	55
36	0.5878	0.7265	1.3764	0.8090	54
37	0.6018	0.7536	1.3270	0.7986	53
38	0.6157	0.7813	1.2799	0.7880	52
39	0.6293	0.8098	1.2349	0.7771	51
40	0.6428	0.8391	1.1918	0.7660	50
41	0.6561	0.8693	1.1504	0.7547	49
42	0.6691	0.9004	1.1106	0.7431	48
43	0.6820	0.9325	1.0724	0.7314	47
44	0.6947	0.9657	1.0355	0.7193	46
45	0.7071	1.0000	1.0000	0.7071	45
°	Cos	Cot	Tan	Sin	°

11. .19 μ coul
12. 6 nt
13. (a) 9×10^7 nt
(b) 9.18×10^9 gf
(c) 1.01×10^4 ton
14. .00196 nt, E
15. (a) 106 coul
(b) 437×10^3 ton
20. 5000 nt/coul, E
21. 2000 nt/coul, S
22. 3.92 μ coul
25. (a) 21.6×10^3 nt/coul, W
(b) 900 nt/coul, E
26. (a) 18,000 nt/coul, W
(b) 0.9 nt/coul, W
27. (a) 18 nt/coul
(b) 2 nt/coul
30. 700 j
31. 80 v, B is higher
31x (a) 6 coul (b) 700 j
32. (a) .025 joul
(b) 1.275 gm
(c) 25,000 v/m, or
25,000 nt/coul
33. (a) 600,000 j
(b) 20,000 nt
40. 3000 kv
41. 18 kv
42. (a) 540 kv, (b) zero
42x (a) Zero, (b) 108,000 v/m
43. (a) 54 kv, 30 kv, 54 kv
(b) 24 kv, A to B
43x (a) -36 kv, 0, +36 kv
44. 72,000 v
54. (a) 400 w
(b) 100 nt, 22.4 lb
57. (a) 80×10^{-17} j
(b) 4.18×10^9 cm/sec
58. (a) 128×10^{-17} j
(b) 16×10^{-16} j
(c) F to A, A to B
59. (a) 128×10^{-17} j
(b) 96×10^{17} j
(c) F to A, B to A
60. 1225 v
60x 4.08×10^{-12} gm
61. 56.5 nt/coul, 0.28 v
70. 4 μfd
71. (a) yes, (b) 800 μcoul
72. (a) .0059 mfd
(b) .0118 mfd
72x .0106 mfd
73. 1250 mmfd
74. (a) 0.4 μ coul
(b) 3200 v, 125 mmfd
75. 300.6 v
76. 0.05 j
77. 36 j
78. (a) 2.5×10^{-6}
(b) 12.5×10^{-6}
80. (a) 0.00003 a, 30 μa
(b) 18.8×10^{13} e/sec
(c) 555.5 min

100. (a) 5 v, (b) 50 w
100 x (a) 0.2v, (b) 10 w
101. 86.4 cal
102. 20 nt, 20 nt/coul
111. 536×10^3 sec
112. w/4 lb
113. 27.1 gm
114. .00068 gm/coul
115. 3.33×10^{-4} gm/coul
116. (a) 0.414 liters
(b) 0.207 liters
117. (a) 2 gm
(b) Vol H = 2 Vol O
118. (a) 113×10^{22}
(b) 9.7×10^{-23} gm
(c) 1.094 gm
(d) 0.000304 gm/coul
119. 17.3 gm
120. 5400 j, 1290 cal
120x (a) 5.94×10^6 j
(b) 6600 w
121. 3.6×10^6 j
122. 6 kw, 8.04 hp
123. (a) 2 μj,
(b) 0.04 μcoul
(c) 8 μj, (d) 6μj
130. 18.3 ohm
131. 60 v
140. (a) 36,000 coul
(b) 3.96×10^6 j
(c) 1100 w, (d) 11 ohm
141. (a) 20.2 ohm, 242 ohm
(b) 5.45 a, 0.45 a
141x 121 ohm, 0.909 a
142. 4.7 a
143. 0.15 w
144. (a) 5 v
145. (a) 100 w
(b) $.05
145x 9 cents
146. (a) 500 w
(b) 14,300 cal
147. Less than 144 w
148. 500 w
150. 4 ohm
150x 5 ohm
151. (a) 100 v
(b) 12,000 j
152. 1 ohm
153. 0.5 a, 1.5 a, 1.3 ohm
154. (a) 6.24 ohm, (b) 29%
155. (a) .5 a, 2 a
(c) 4 to 1
156. (a) 220 watt iron
160. 4.62 ohm
161. (a) 2.4 a, 0.8 a, 0.4 a
(b) 2.4 v (c) 2/3 ohm
161x 1.25 ohm, 10 a, 2 a
(a) 1.23 ohm, (b) 5 a
164. (a) 101 ohm, (b) 3.96 ohm
165. 11 ohm and 1 ohm
166. (a) 0.0999% (b) 100,000%
180. 0.0001 ohm

181. (a) 9 ohm, (b) 0.5 a
182. 0.5 ohm
190. 2320 ohm
191. 4200 ohm
192. (a) 200 ohm, (b) 400 ohm
(c) 39.8 volts
195. 20.06 volts
200. (a) 4 v battery charged,
20 v battery discharged
(b) 15 v, B higher
(c) 9 v, B higher
201. Discharging current >3a
210. (a) 0.5 a, (b) 5.5 v
(c) A=0,B=5.5,C=5.5,D=1.5
(d) A=0,B=6,C=0,D=0
211. (a) 15 a (b) 0.75 v
(c) 338 j (d) 338 j
212. 2.5 a, 0.2 ohm
213. (a) 200 a (b) 0, (c) Heat
214. 1 ohm total of both wires
215. (a) 1.364 v, (b) 1.485 v
216. (a) 119 ohm, 1 ohm
(b) 6/119 a, (c) 6 amp
220. 0.5 amp
221. 8 v, 6 ohm
230. (a) 4 a from left to right
(b) 2/3 ohm
230x (a) 4 v, A is higher
(b) 1 a from T to A
(c) 2 ohms
231. 0.4 amp
240. (a) 150 ohm, (b) 0.012 a
(c) yes, (d) yes
241. 1/2 ohm, 222 ohm
242. 5 ohm, 15 v
250. 1 ohm
251. 0.045 cm
251x .11 cm
253. 100 ohm C.M./ft
253x 8.0 ft
255. (a) 80 ohm C.M./ft
(b) same
256. (a) 1.64 times
(b) 1.28 times
260. 0.00294/°C
261. 16.4 ohms
270. (a) 1 ohm
271. (a) 1 ohm (b) 25 cm (c) No
274. (a) From T'
(b) From T
(c) Zero, (d) From T'
(e) Zero, (f) From T
275. 0.5 v, 5 v
276. More than 1.37
277. 5 a
280. 9.6 mv
281. (b) k = .8
290. (b) 96 v (c) 10.6 ohm
300. 0.0049 web/m^2, 49 gauss
301. 0.003 nt (.306 gf),
south, 30° below horizontal
302. 0.0026 nt, W
303. 0.0015 nt, E

310. (a) 0.0008 nt, E
(b) 0.00004 nt m
or .408 gf cm
320. Vertically up
321. (a) 0.19 cm, (b) clockwise
330. (a) 0.5 gauss, 37° N of W
(b) 37° N of W
331. (a) 0.173 gauss, (b) S pole
334. 0.363 gauss
335. 0.0519 nt (5.29 gf)
340. (a) 25 x 10^{-6} lines (web)
(b) 40,000 m^2
(c) 25 lines (microweb)
341. (a) 0.392 lines (web)
(b) 392,000 lines (microweb)
(c) 15 nt (1.53 kgf)
342. (a) 25.9 lines (web)
(b) 34.6 lines (microweb)
350. (a) 3.9 x 10^{-12} web/m^2, W
(b) more than 3100 times
351. At P, 3 microweb/m^2, S
At R, 1.5 microweb/m^2,
At T, Zero
352. (a) 47.1 microweb/m^2, S
(b) Decreased
360. 4700 microweb/m^2
361. 94.0 microweb/m^2
362. 26.7 microweb/m^2, down
370. (a) .346 gauss
(b) .688 amp
(c) up
371. (a) 4.24 x 10^{-3} amp
(b) .2 gauss
390. 8 microweb/m^2 (.08 gauss)
391. (a) 400 amp North
400. 4,460 microweb
401. (a) .241 web/m^2
(b) 9.64 x 10^{-6} seb
9.64 microweb
402. (a) 3.14 amp
(b) 1.20 microweb
403. 3.98 amp
451. (a) .0001 web/m^2
(b) 0045 nt, up
452. (a) .0003 web/m^2
(b) 0045 nt, down
490. .03 v
491. (a) 3.02 microweb
(b) 4.52 microvolt
500. (a) .25 v, (b) 20 nt
510. 8.5 microweb
511. .2 microweb
512. 20 microweb/m^2
513. 40 microcoul
514. 17.3 microcoul
515. (a) 10 microweb
(b) .05 web/m^2
530. (a) 60,000 microweb/m^2
(b) 12o
(c) 36,000 microweb/m^2
531. (a) 100,000 microweb/m^2
(b) 16.7, (c) 6 microweb
532. 12,000 microweb

534. (a) 1000
(b) 509.5 microweb
540. 8 x 10^{-6} web
541. 3.2 gauss
562. 50 w
563. 90 v
564. (a) 136 v, (b) 4 ohm
(c) 4602 w, (d) 78.2%
565. (a) 770 w, (b) 100 v
(c) -2%, +20%, +17.6%
566. (a) 4 a, (b) 6 a, (c) 2 a
567. (a). 12 a, (b) 6a, (c) 6 a
568. (a) 4 a, (b) 6 a, (c) 10 a
569. 9 v
580. (a) 0.05 sec, (b) 20 sec
582. 1.6 mfd
583. (a) 500 ohm, (b) .05 a
583x (a) 2000 ohm, (b) .012 a
590. 0.15 ma
591. 2.88 ma
592. (a) 0.5 mfd
(b) 40 μ a, B to F, (c) zero
600. 0.02 hy
601. 120 v
610. 0.04 hy, 40 mhy
611. 5000 j
620. A, 0.012 hy, 4 ohm
B, 0.003 hy, 6 ohm
621. (a) 12 a/sec, (b) 3 a
(c) 9 a/sec
750. (a) 0, 50, 100, 50, 0, -100
(b) + in I and II,
- in III and IV
(c) 0.50 v
751. (e) 40π rad/sec
(f) 86.6 v each
760. (a) 4 a, 5.6 a
(b) 112 v, 80 v
761. 155 v
762. 50%
780. 0.0035 hy
782. (a) 1.99 a, (b) 0.12 a
790. (a) 1.36 a, (b) 136 a
791. 99 mfd
795. (a) 2200 ohm
(b) 11 ohm
796. (a) 2, 0, infinity
(b) 2, 0.5, 400
(c) 2, 6000, .083
797. (a) 5, infinity, 0
(b) 5, 20, 1/40
(c) 5, 1/600, 300
801. 10 v, 53°
802. 13.8 v, 270°
803. -30°, 208 v
810. 30 v, 37.7 v, 48.1 v, 52°
811. 21.2 v, 26.6 v, 34 v
830. (a) 22 a, (b) .0106 hy
(c) 1.64 a, (d) 36.6 a
831. 50 ohm, 0796 hy
832. 10.69 ohm

840. (a) 3.77 ohm, 13.26 ohm
5 ohm, 10.7 ohm
(b) 7.54 v, 26.5 v, 10 v
841. (a) 200 ohm, (b) 5 ohm
(c) 2 amp, (4) 400 v
843. (a) 92.4 v (b) 20.6 ohm
870. 120 w
880. (a) 0.5 a, (b) 300 turns
881. (a) 7500 turns, (b) 3 a
882. (a) 1800 kw
(b) 50,000,000 kw

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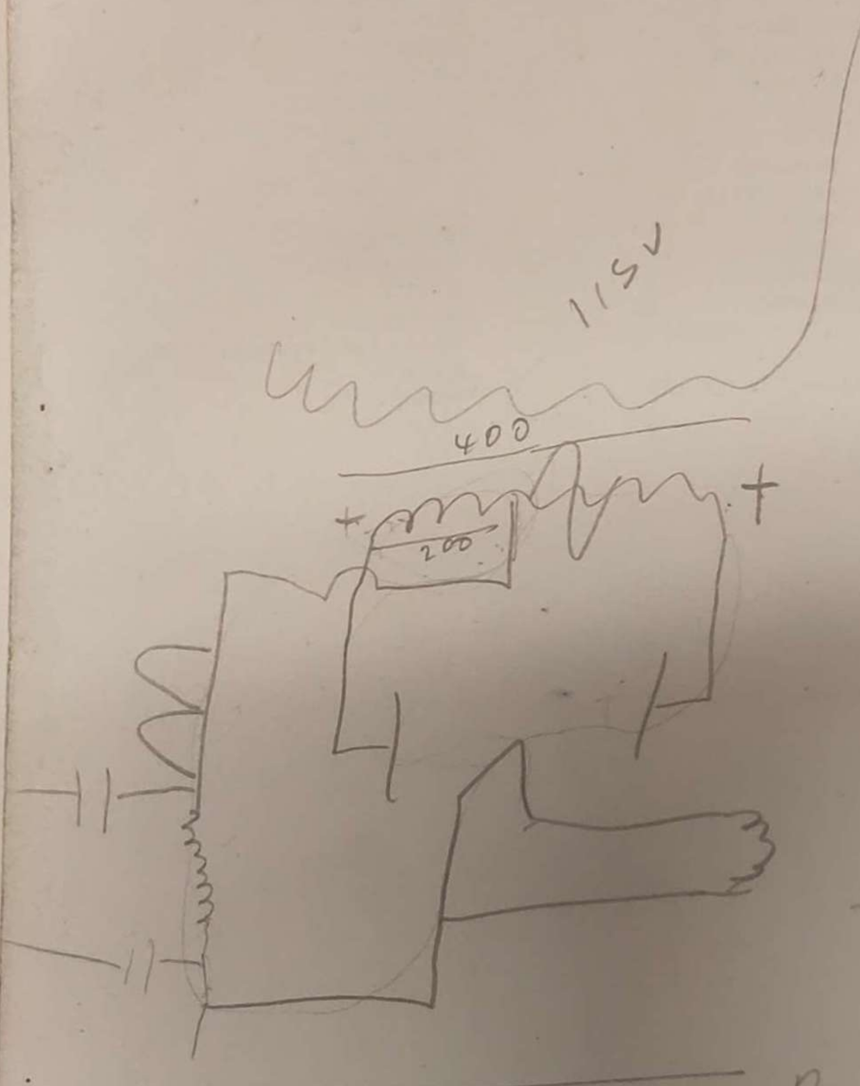

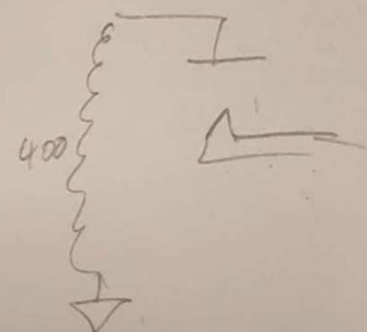

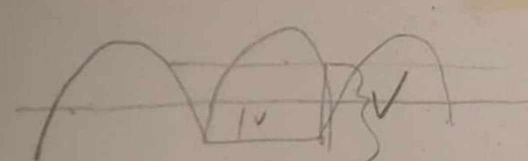

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